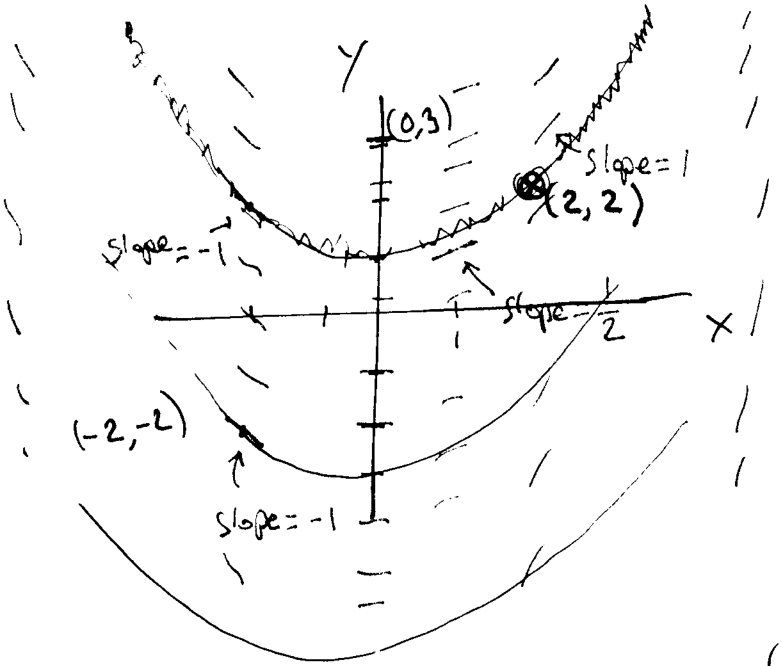
9.1 (Loose ends) Slope fields, word problems

<u>Slope fields</u>: a way to visualize differential equations

<u>ex</u>: $\frac{dy}{dx} = \frac{1}{2}x$ ← <u>In words</u>: There is some function $y = f(x)$ whose derivative (slope of its tangent lines) is $\frac{1}{2}x$.



when $(x,y) = (1,1)$,

$$\frac{dy}{dx} = \text{slope} = \frac{1}{2}(1) = \frac{1}{2}$$

(x,y)	slope $= \frac{dy}{dx}$
$(1,1)$	$\frac{1}{2}$
$(-2,-2)$	$\frac{dy}{dx} = \frac{1}{2}(-2) = -1$
$(-2,2)$	$\frac{dy}{dx} = \frac{1}{2}(-2) = -1$
$(0,3)$	$\frac{dy}{dx} = \frac{1}{2}(0) = 0$

$$\int dy = \int \frac{1}{2} x \, dx$$

$$y = \frac{1}{4}x^2 + C$$

what is C if $y = 2$ when $x = 2$.

$$2 = \frac{1}{4}(2)^2 + C$$

$$2 = 1 + C \Rightarrow C = 1$$

$$\boxed{y = \frac{1}{4}x^2 + 1}$$

The particular solution satisfying the diff. eqn. <u>and</u> the initial condition.

ex 1: You save \$2000 per year (continuously) by putting in a pillow case. Find a Diff Equ to model this. Let t = time (years) and $y(t)$ = balance (dollars).

$$\frac{dy}{dt} = \text{rate of change of } y \text{ with respect to } t = 2000 \text{ dollars/year}$$

Solution: $\int dy = \int 2000\, dt$

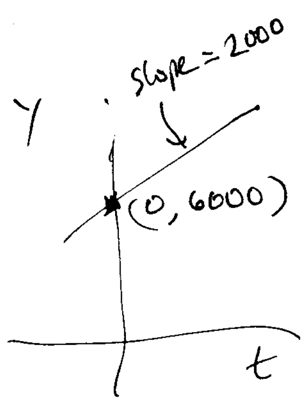
$$y = 2000t + C$$

added info: You start with (at $t=0$) 6000 dollars.

so $y(0) = 2000(0) + C = 6000$

so $C = 6000$

$$\boxed{y(t) = 2000t + 6000}$$



Ex 2: You have \$6000 in a bank which pay 10% per year "compounded continuously".

$$\frac{dy}{dt} = 0.10\,y \quad , \text{ and } \quad y(0) = 6000$$

instantaneous rate of change of bank balance

directly is proportional to your current balance

solution:

$$\int \frac{dy}{y} = \int 0.10\,dt$$

$$\ln y = 0.10t + C$$

what is C? Use: when $t=0$, $y = 6000$.

$$\ln 6000 = 0.10(0) + C = C$$

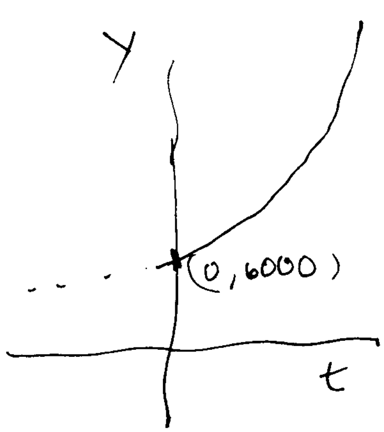
So

$$\ln y = 0.10t + \ln 6000$$

$$e^{\ln y} = e^{0.10t + \ln 6000}$$

$$y = e^{\ln 6000} \cdot e^{0.10t}$$

$$\boxed{y = 6000\,e^{0.10t}}$$



59) t = time (years)

$y(t)$ = balance (thousands of dollars)

a) $y(0) = 6$ i.e. you start with \$6000

$$\frac{dy}{dt} = 0.10\,y + 3 \quad ; \quad y(0) = 6$$ ← part b)

rate of change of balance

interest at 10% of balance per year

contributions of \$3000 per year

c) Solve: $\frac{dy}{dt} = 0.10\,y + 3 = 0.10\,(y + 30)$

$$\int \frac{dy}{y+30} = \int 0.10\,dt$$

$$\ln|y+30| = 0.10t + C$$

$$e^{\ln|y+30|} = e^{0.10t + C}$$

$$|y+30| = e^{C} \cdot e^{0.10t}$$

$$y + 30 = \pm e^{C} \cdot e^{0.10t} = C_1 e^{0.10t}$$ where $\pm e^{C} = C_1$

$$y = C_1 e^{0.10t} - 30$$

when $t=0$, $y=6$: $6 = C_1 e^{0} - 30$

$$6 = C_1 - 30 \Rightarrow C_1 = 36$$

$$\boxed{y(t) = 36\,e^{0.10t} - 30}$$

d) what is $y(25)$?

$$y(25) = 36\, e^{0.10(25)} - 30$$

$$= 36\, e^{2.5} - 30$$

$$= 36\,(12.1825) - 30$$

$$= 408.56978$$

You have \$408,569.78 .