Warmup

9.3 24) $xy' + 4y = 10x$; $y(1) = 0$

Divide by x: $y' + \frac{4}{x}y = 10$ is standard form for a 1^{st}-order linear D.E.

$$P(x) = \frac{4}{x}$$

$$\int p\,dx = \int \frac{4}{x}\,dx = 4\ln x = \ln x^4$$

$$e^{\int p\,dx} = e^{\ln x^4} = x^4$$

$$x^4 y' + x^4 \cdot \frac{4}{x} y = 10x^4$$

$$x^4 y' + 4x^3 y = 10x^4$$

$$\frac{d}{dx}\left[x^4 y\right] = 10x^4$$

product rule backwards

$$x^4 y = \int 10x^4\,dx$$

$$x^4 y = 2x^5 + C$$

$$y = \frac{2x^5}{x^4} + \frac{C}{x^4}$$

$$y(x) = 2x + \frac{C}{x^4}$$

$$0 = y(1) = 2 + C \Rightarrow C = -2$$

$$\boxed{y(x) = 2x - \frac{2}{x^4}}$$

Survey of Chapter 10

10.1 Geometric series

ex①. $3 + 3\cdot 2 + 3\cdot 2^2 + 3\cdot 2^3 + 3\cdot 2^4 + 3\cdot 2^5$
$= 3 + 6 + 12 + 24 + 48 + 96$

ex②: $1000 + 1000(1.02) + 1000(1.02)^2 + 1000(1.02)^3$

General form: $a + ar + ar^2 + ar^3 + \dots + ar^{n-1} = S_n$

a = first term
r = common ratio
n = number of terms

S_n = sum of this finite geometric series

$$S_n = a + ar + ar^2 + \dots + ar^{n-2} + ar^{n-1}$$
$$rS_n = ar + ar^2 + \dots + ar^{n-2} + ar^{n-1} + ar^n$$

subtract: $S_n - rS_n = a - ar^n$

$$(1-r)S_n = a(1-r^n)$$

$$\boxed{S_n = a\,\frac{1-r^n}{1-r} = a\,\frac{r^n-1}{r-1}}$$

ex ①: $S_6 = 3\cdot\dfrac{2^6-1}{2-1} = 3\cdot\dfrac{(64-1)}{1} = 3\cdot 63 = 189$

ex②: $S_4 = 1000\cdot\left(\dfrac{1.02^4-1}{1.02-1}\right) = 1000\left(\dfrac{1.0824-1}{0.02}\right)$

$= 1000\left(\dfrac{0.08243216}{0.02}\right) = 1000(4.121608)$
$= 4{,}121.61$

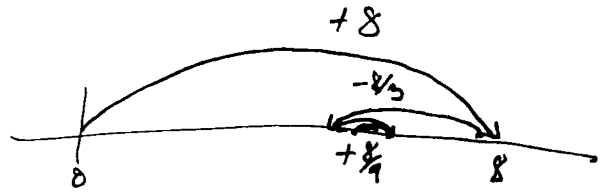
ex (Infinite geometric series)

$$8 - \frac{8}{3} + \frac{8}{9} - \frac{8}{27} + \frac{8}{81} - \cdots$$

$a = 8$

$r = -\frac{1}{3}$

$n = \infty$



Jumping flea

where on the ruler does the flea end up?

$$S_\infty = a \cdot \frac{1 - \cancel{r}^{\infty}}{1 - r} = 8 \cdot \frac{1 - \left(-\frac{1}{3}\right)^\infty}{1 - \left(-\frac{1}{3}\right)}$$

$$= 8 \cdot \frac{1 - 0}{1 - \left(-\frac{1}{3}\right)} = \frac{8}{4/3} \cdot \frac{3}{3} = \frac{8 \cdot 3}{4}$$

$$= 6$$

Answer: 6 inch mark

Fact: If $|r| < 1$ then

$$\boxed{S_\infty = \frac{a}{1 - r}}$$

Intro to 10.2 Taylor Polynomials

The Taylor Polynomial at $x=0$ of $f(x)$ is

$$p_n(x) = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + \cdots + a_n x^n$$

where
$$a_0 = f(0)$$
$$a_1 = \frac{f'(0)}{1!}$$
$$a_2 = \frac{f''(0)}{2!}$$
$$a_3 = f'''(0)/3!$$
$$\vdots$$
$$a_n = f^{(n)}(0)/n!$$

ex: $f(x) = \cos x$

n	$f^{(n)}(x)$	$f^{(n)}(0)$	$f^{(n)}(0)/n!$
0	$\cos x$	1	1
1	$-\sin x$	0	0
2	$-\cos x$	-1	$-1/2! = -\frac{1}{2}$
3	$\sin x$	0	0
4	$\cos x$	1	$1/4! = \frac{1}{24}$
5	$-\sin x$	0	0
6	$-\cos x$	-1	$-1/6! = \frac{-1}{720}$

note: $0! = 1$

$$P_6(x) = 1 - \frac{1}{2}x^2 + \frac{1}{24}x^4 - \frac{1}{720}x^6$$