Review for Final Part 2

Maybe one question from 10.1 Geometric Series

10.1 15)

$$3 + 3\cdot 4 + 3\cdot 4^2 + 3\cdot 4^3 + 3\cdot 4^4 + 3\cdot 4^5$$

S_n = sum of first n terms of a geometric

$$\boxed{S_n = a\,\frac{1-r^n}{1-r}}$$

a = first term

r = common ratio

n = number of terms
= first power of n left out of the series

15 cont'd) $a = 3$

$r = 4$

$n = 6$

so $S_n = 3\cdot\frac{1-4^6}{1-4} = 3\cdot\frac{1-4096}{1-4}$

$= 3\cdot\frac{-4095}{-3} = \frac{-4095}{-1} = 4095$

10.1

21) $2 - \frac{2}{3} + \frac{2}{9} - \frac{2}{27} + \frac{2}{81} - \cdots$

$$\boxed{S = \frac{a}{1-r}}$$

where a = 1st term
r = common ratio

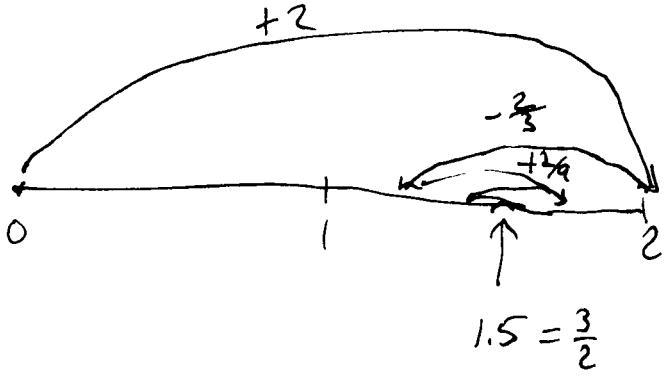
But this "converges" only if $|r| < 1$.

21 cont'd) $a = 2$

$r = -\frac{1}{3}$

So $S = \frac{2}{1-\left(-\frac{1}{3}\right)} = \frac{2}{1+\frac{1}{3}} = \frac{2}{\frac{4}{3}} = 2 \div \frac{4}{3}$

$= \frac{\cancel{2}^{1}}{1} \cdot \frac{3}{\cancel{4}_{2}} = \boxed{\frac{3}{2}}$



Allowed notes

- Blue sheet of integration formulas (inside back cover)
- One side of one page of handwritten notes (your handwriting)

Overview

- 5.6 Substitution (indirectly)
- Chap 6: Integration techniques
- Chap 7: Calculus of two or three variables
- Chap 8: sine and cosine
- 9.1: Diff. eqn.: separation of variables
- 10.1: Geometric series

More detail

example: use substitution for this integral

(1) $\int \frac{\cos x}{\sin x}\,dx$ answer: $u = \sin x$, $du = \cos x\,dx$

$$= \int \frac{du}{u} = \ln|u| + C = \ln|\sin x| + C$$

(2) $\int \cos x\, e^{3\sin x}\,dx = \frac{1}{3}\int e^{3\sin x}\cdot 3\cos x\,dx$

$u = 3\sin x$, $du = 3\cos x\,dx$

$$= \frac{1}{3}\int e^u\,du$$

$$= \frac{1}{3}e^u + C = \frac{1}{3}e^{3\sin x} + C$$

6.1 Integration by parts (at least one)
repeated (tabular) integration by parts

6.2 Integration using table (blue sheet) one.

6.3 Improper integer: may converge or diverge

ex: $\int_4^\infty \frac{1}{\sqrt{x^3}}\,dx = \int_4^\infty \frac{1}{x^{3/2}}\,dx$ (abuse notation)

$= \int_4^\infty x^{-3/2}\,dx = -2x^{-1/2}\Big|_4^\infty$

$= -\frac{2}{\sqrt{\infty}} + \frac{2}{\sqrt{4}} = -0 + \frac{2}{2} = \underline{1}$

ex: $\int_1^\infty \frac{1}{\sqrt{x}}\,dx = \int_1^\infty x^{-1/2}\,dx = 2x^{1/2}\Big|_1^\infty$

$= 2\sqrt{\infty} - 2\sqrt{1} = \infty$ Divergent.

6.4 Simpson rule etc - skip

7.1 Intro to functions of two variables – no direct question

7.2 Partial derivatives – including second derivatives:

$f_{xx}, f_{xy}, f_{yx}, f_{yy}$

and evaluate $f_x(3,5)$.

ex: $f(x,y) = x e^{3xy}$. Find $f_x(3,5)$

product rule → $f_x(x,y) = 1 \cdot e^{3xy} + x \cdot e^{3xy} \cdot \underbrace{\frac{\partial}{\partial x}[3xy]}_{3y}$ ← chain rule

$= e^{3xy} + 3xy\, e^{3xy}$

$f_x(3,5) = e^{45} + 45 e^{45}$

7.3 Max/min of a function of two variables (word problem?)

Put in your notes: D-test $D = \begin{vmatrix} f_{xx} & f_{xy} \\ f_{xy} & f_{yy} \end{vmatrix}$

$= f_{xx} f_{yy} - [f_{xy}]^2$

at a critical point (where $\begin{cases} f_x(x,y)=0 \\ f_y(x,y)=0 \end{cases}$)

	$f_{xx}>0$	$f_{xx}<0$
$D>0$	rel min	rel max
$D<0$	saddle point	
$D=0$	¿Quién sabe?	

7.4 we skipped

7.5 Lagrange multiplier (most likely example of a word problem)

put into notes:

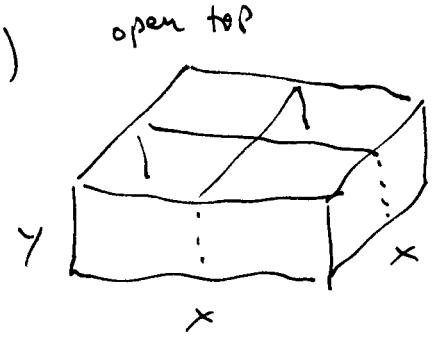
Objective function: (thing to maximize or minimize) $f(x,y)$

Constraint: $g(x,y) = 0$

To solve the constrained optimization problem,
Find the critical points of

$$F(x,y,\lambda) = f(x,y) + \lambda\, g(x,y)$$

p555 48)



Volume = 576 in^3

minize the material (in^2) used.

$$\text{material} = f(x,y) = x^2 + 6xy$$

(x^2: base; $6xy$: vertical sides or dividers)

constraint: $x^2y = 576$

$$g(x,y) = x^2y - 576 = 0$$

$$F(x,y,\lambda) = x^2 + 6xy + \lambda(x^2y - 576)$$

$0 = F_x$

$0 = F_y$

$0 = F_\lambda$

7.6 Total differential

- Find df e.g. $f(x,y,z) = x^2 y e^z$ Find df
- Use in approximation (maybe)

$$df = \frac{\partial f}{\partial x}dx + \frac{\partial f}{\partial y}dy + \frac{\partial f}{\partial z}dz$$

$$= 2xye^z dx + x^2 e^z dy + x^2 y e^z dz$$

7.7. Double integral one already set up iterated integral.

e.g. $\int_1^3 \int_0^2 x^2 y \, dx\, dy$

8.1 } no direct questions
8.2 }

8.3 - Derivative of sin, cos

8.4 Integral of sin, cos

9.1 Separable $\frac{dy}{dx} = -\frac{x}{y}$, $y(3) = 4$.

10.1 [already mentioned]