15 June 2015

Math 150 Chap 7 - Techniques of integration

"Modified" Basic Integral Formulas

example: $\int \cos x \, dx = \sin x + C$ (Basic formula)

$\int \cos(3x+5)\,dx = \frac{1}{3}\sin(3x+5) + C$ (modified basic)

ex: $\int \frac{1}{x}\,dx = \ln|x| + C$

$\int \frac{1}{5x-13}\,dx = \frac{1}{5}\ln|5x-13| + C$

Basic	Modified Basic
$\int x^n\,dx = \frac{x^{n+1}}{n+1} + C$	$\int (ax+b)^n\,dx = \frac{1}{a}\frac{(ax+b)^{n+1}}{n+1} + C$
$\int e^x\,dx = e^x + C$	$\int e^{ax+b}\,dx = \frac{1}{a}e^{ax+b} + C$
$\int \sin x\,dx = -\cos x + C$	$\int \sin(ax+b)\,dx = -\frac{1}{a}\cos(ax+b) + C$
$\int f(x)\,dx = F(x) + C$	$\int f(ax+b)\,dx = \frac{1}{a}F(ax+b) + C$

Reason: $\frac{d}{dx}\cos(ax+b) = -\sin(ax+b)\cdot\frac{d}{dx}[ax+b]$
$= (-\sin(ax+b))\cdot a$

so $\frac{d}{dx}\left[-\frac{1}{a}\cos(ax+b)\right] = \sin(ax+b)$

so $\int \sin(ax+b)\,dx = -\frac{1}{a}\cos(ax+b) + C$

Review of integration by substitution

Try

1. Basic formula.
2. modified basic formula
3. Algebra.
4. Substitution

ex: $\int 2x\sqrt{x^2+1}\,dx$

$u = x^2+1$
$du = 2x\,dx$

$$= \int \sqrt{u}\,du$$
$$= \int u^{1/2}\,du$$
$$= \frac{2}{3}u^{3/2} + C$$
$$= \frac{2}{3}(x^2+1)^{3/2} + C$$

ex: ["In-line" substitution]

$$\int 3\sin^2 x\cos x\,dx = \int 3u^2\,du$$
$$= u^3 + C$$
$$= \sin^3 x + C$$

$u = \sin x$
$du = \cos x\,dx$

"In-line" substitution [Don't do this unless you know what you're doing.]

$$\int 3\sin^2 x\cos x\,dx = \int 3\sin^2 x\,d(\sin x)$$
$$= \sin^3 x + C$$

Idea: Recognize that
$\cos x\,dx = d(\sin x)$

Substitution in definite integrals

ex: $$\int_{x=0}^{x=3} x e^{x^2}\,dx = \frac{1}{2}\int_{u=0}^{u=9} e^u\,du$$

$u = x^2$

$du = 2x\,dx$

$\frac{1}{2}du = x\,dx$

$x=0 \Leftrightarrow u = 0^2 = 0$

$x=3 \Leftrightarrow u = 3^2 = 9$

$$= \frac{1}{2}e^u \Big]_{u=0}^{u=9}$$

$$= \frac{1}{2}e^9 - \frac{1}{2}e^0 = \boxed{\frac{1}{2}e^9 - \frac{1}{2}}$$