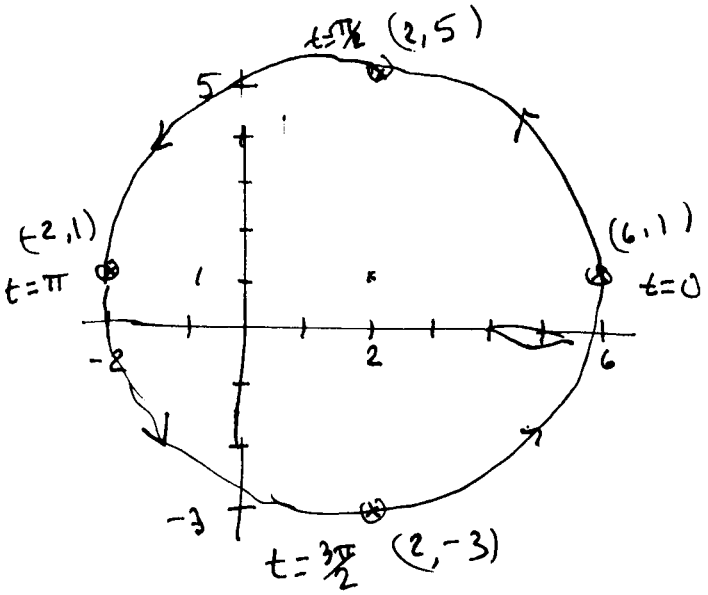
10.1 Curves Defined by Parametric Equations

ex: $\begin{cases} x = 2 + 4\cos t \\ y = 1 + 4\sin t \end{cases}$ $\quad 0 \le t \le 2\pi$

t	x	y
0	$2+4=6$	$1+0=1$
$\pi/2$	$2+0=2$	$1+4=5$
π	$2-4=-2$	$1+0=1$
$3\pi/2$	$2+0=2$	$1-4=-3$
2π	6	1



Is this genuinely a circle?

Let's "eliminate the parameter" i.e. go from two equations in x, y, and t to one equation in x and y.

$$x - 2 = 4\cos t \Rightarrow (x-2)^2 = 16\cos^2 t$$
$$y - 1 = 4\sin t \qquad (y-1)^2 = 16\sin^2 t$$

$$(x-2)^2 + (y-1)^2 = 16(\underbrace{\cos^2 t + \sin^2 t}_{1})$$

A circle of radius $= 4$ and center $= (2,1)$ $\rightarrow$ $\boxed{(x-2)^2 + (y-1)^2 = 16}$

Remark: In eliminating the parameter, we lose the counter-clockwise orientation.

7th ed. 13) $\begin{cases} x = \sin t \\ y = \csc t \end{cases}$ $\quad 0 < t < \pi/2$

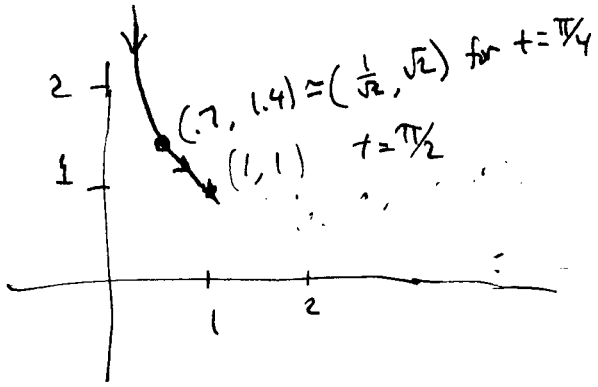
a) Eliminate the parameter to find a Cartesian equation of the curve.

b) Sketch the curve.

a) Since $x = \sin t$ and $y = \frac{1}{\sin t} \Rightarrow y = \frac{1}{x}$

b)

t	(x, y)
0	$(0, \infty)$
$\pi/4$	$\left(\frac{\sqrt{2}}{2}, \sqrt{2}\right) \simeq (.7, 1.4)$
$\pi/2$	$(1, 1)$

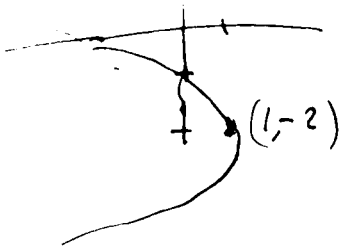


7) $\begin{cases} x = 1 - t^2 \\ y = t - 2 \end{cases}$ $\quad -2 \le t \le 2$

$\Rightarrow t = y + 2$ Then sub into 1st equation

$$\boxed{x = 1 - (y+2)^2}$$ a horizontal parabola

vertex: $(x, y) = (1, -2)$



t	(x, y)
-2	$(-3, -4)$
0	$(1, -2)$
2	$(-3, 0)$