

Summary of Tests for Series

Test	Series	Convergence or Divergence
n th-term	$\sum a_n$	Diverges if $\lim_{n \rightarrow \infty} a_n \neq 0$
Geometric series	$\sum_{n=1}^{\infty} ar^{n-1}$	Converges with sum $S = \frac{a}{1-r}$ if $ r < 1$. Diverges if $ r \geq 1$.
Telescoping series	$\sum_{n=1}^{\infty} (b_n - b_{n+1})$	Converges with sum $S = b_1 - L$ if $\lim_{n \rightarrow \infty} b_n = L$.
p -series	$\sum_{n=1}^{\infty} \frac{1}{n^p}$	Converges if $p > 1$. Diverges if $p \leq 1$.
Integral	$\sum_{n=1}^{\infty} a_n$ $a_n = f(n)$	Converges if $\int_1^{\infty} f(x) dx$ converges. Diverges if $\int_1^{\infty} f(x) dx$ diverges. Remainder: $ S - S_n \leq \int_n^{\infty} f(x) dx$ if convergent.
Comparison	$\sum a_n, \sum b_n$ $a_n > 0, b_n > 0$	If $\sum b_n$ converges and $a_n \leq b_n$, then $\sum a_n$ converges. If $\sum b_n$ diverges and $a_n \geq b_n$, then $\sum a_n$ diverges. If $\lim_{n \rightarrow \infty} (a_n/b_n) = c > 0$, both series converge or both diverge.
Ratio	$\sum a_n$	If $\lim_{n \rightarrow \infty} \left \frac{a_{n+1}}{a_n} \right = L$ (or ∞), converges absolutely if $L < 1$; diverges if $L > 1$ (or ∞).
Root	$\sum a_n$	If $\lim_{n \rightarrow \infty} \sqrt[n]{ a_n } = L$ (or ∞), converges absolutely if $L < 1$; diverges if $L > 1$ (or ∞).
Alternating series	$\sum (-1)^n a_n$, $a_n > 0$	Converges if $a_k \geq a_{k+1}$ for every k ; and $\lim_{n \rightarrow \infty} a_n = 0$. Remainder: $ S - S_n \leq a_{n+1}$
$\sum a_n $	$\sum a_n$	If $\sum a_n $ converges, then $\sum a_n$ converges.