## **Power Series for Several Elementary Functions**

Function  

$$\frac{1}{x} = 1 - (x - 1) + (x - 1)^{2} - (x - 1)^{3} + \dots + (-1)^{n} (x - 1)^{n} + \dots \qquad 0 < x < 2$$
Interval of Convergence  
 $0 < x < 2$ 

$$\frac{1}{1+x} = 1 - x + x^2 - x^3 + \dots + (-1)^n x^n + \dots -1 < x < 1$$

$$\ln x = (x-1) - \frac{(x-1)^2}{2} + \frac{(x-1)^3}{3} - \dots + (-1)^{n-1} \frac{(x-1)^n}{n} + \dots \qquad 0 < x \le 2$$

$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \dots + (-1)^{n-1} \frac{x^n}{n} + \dots$$
  $-1 < x \le 1$ 

$$e^{x} = 1 + x + \frac{x^{2}}{2!} + \frac{x^{3}}{3!} + \dots + \frac{x^{n}}{n!} + \dots$$
  $-\infty < x < \infty$ 

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \dots + (-1)^n \frac{x^{2n+1}}{(2n+1)!} + \dots -\infty < x < \infty$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \dots + (-1)^n \frac{x^{2n}}{(2n)!} + \dots -\infty < x < \infty$$

$$\tan^{-1} x = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \dots + (-1)^n \frac{x^{2n+1}}{2n+1} + \dots -1 \le x \le 1$$

$$\sin^{-1} x = x + \frac{x^3}{2 \cdot 3} + \frac{1 \cdot 3x^5}{2 \cdot 4 \cdot 5} + \frac{1 \cdot 3 \cdot 5x^7}{2 \cdot 4 \cdot 6 \cdot 7} + \dots + \frac{(2n)! x^{2n+1}}{(2^n n!)^2 (2n+1)} + \dots -1 \le x \le 1$$

$$(1+x)^{k} = 1 + kx + \frac{k(k-1)x^{2}}{2!} + \frac{k(k-1)(k-2)x^{3}}{3!} + \dots + \frac{k(k-1)\dots(k-n+1)x^{n}}{n!} + \dots - 1 < x < 1*$$
  
\*The binomial series may converge at  $x = \pm 1$  for some values of k.