1. Let \( z = f(x, y) = e^{2x} \sin 3y \).
   a. Find the total differential \( dz \), that is \( dz = f_x(x, y) \, dx + f_y(x, y) \, dy \).
   
   \[
   dz = \frac{\partial z}{\partial x} \, dx + \frac{\partial z}{\partial y} \, dy = \left[ 2e^{2x} \sin 3y \, dx + 3e^{2x} \cos 3y \, dy \right]
   \]

   b. Use \( dz \) to approximate \( \Delta z = f(0.03, 0.04) - f(0, 0) \) by taking \( (x, y) = (0, 0) \) and
   \( (\Delta x, \Delta y) = (dx, dy) = (0.03, 0.04) \) when \( (x, y) = (0, 0) \) and \( (dx, dy) = (0.03, 0.04) \).

   \[
   \Delta z = (2e^0 \sin 0.03) \cdot 0.03 + (3e^0 \cos 0.03) \cdot 0.04
   \]

   \[
   = 3(0.04) = 0.12
   \]

   [This approximates \( \Delta z = e^{2(0.03)} \sin 3(0.04) - 0 = 0.1271 \).]

2. The radius of a right circular cylinder is increasing at 6 cm per second, and the height is increasing at 4 cm per second. Use a chain rule to determine the rate of change of the volume when the radius is 12 cm and the height is 36 cm.

   To find: \( \frac{dV}{dt} \), given that \( (r, h) = (36 \text{ cm}, 12 \text{ cm}) \) and \( \left( \frac{dr}{dt}, \frac{dh}{dt} \right) = (4 \text{ cm/s}, 6 \text{ cm/s}) \).

   Volume of a cylinder = \( \pi r^2 h \) \( \therefore \)

   \[
   \frac{dV}{dt} = \frac{dV}{dr} \frac{dr}{dt} + \frac{dV}{dh} \frac{dh}{dt} = \pi r^2 \frac{dh}{dt} + 2\pi rh \frac{dr}{dt}
   \]

   \[
   = \pi \left[ (12)^2 \cdot 4 + 2 \cdot 12 \cdot 36 \cdot 6 \right] = \pi \left( 12^2 \right) \left[ 4 + 36 \right]
   \]

   \[
   = \boxed{5760 \pi \text{ cm}^3/\text{s}} \approx 18,095.6 \text{ cm}^3/\text{s}
   \]

3. Suppose that \( x = g(y, z) \) is implicitly defined by the equation \( x^3 - 2xy + z^3 + 7y + 6 = 0 \). Find \( \frac{dx}{dy} \) and \( \frac{dx}{dz} \).

   Let \( F(x, y, z) = x^3 - 2xy + z^3 + 7y + 6 \).

   Then \( g_y(y, z) = \frac{\partial x}{\partial y} = -\frac{F_y(x, y, z)}{F_x(x, y, z)} = \boxed{-\frac{-2x + 7}{3x^2 - 2y}} \)

   and \( g_z(y, z) = \frac{\partial x}{\partial z} = -\frac{F_z(x, y, z)}{F_x(x, y, z)} = \boxed{-\frac{3z^2}{3x^2 - 2y}} \)