Some Tips on Parametrizing Curves

1. For plane curves of the form $y = g(x)$ or $x = h(y)$, take the parameter t to be equal to the *independent* variable.

Example 1: Parametrize the graph of $y^2 = x$ from (1, 1) to (4, 2).

Note this has the form $x = h(y) = y^2$. Take $y(t) = t$. Then $x = h(t) = t^2$, where $1 \le t \le 2$.

2. For plane curves of the form $r = g(\theta)$ in polar coordinates, let $t = \theta$, and let $x(t) = g(t)\cos t$, $y(t) = g(t)\sin t$.

Example 2: Parametrize one loop of the graph $r = \sin 2\theta$, $0 \le t \le \frac{\pi}{2}$.

Here $r = g(\theta) = \sin 2\theta$, so let $x(t) = \sin 2t \cos t$, $y(t) = \sin 2t \sin t$, $0 \le t \le \frac{\pi}{2}$.

3. For lines in space (or in the plane) from $P = (x_1, y_1, z_1)$ to $Q = (x_2, y_2, z_2)$, let $x(t) = x_1 + (x_2 - x_1)t$, $y(t) = y_1 + (y_2 - y_1)t$, $z(t) = z_1 + (z_2 - z_1)t$, $0 \le t \le 1$.

Example 3: Parametrize the line from (1, 2) to (– 1, 3).

Here we can take $x(t) = 1 - 2t$, $y(t) = 2 + t$, $0 \le t \le 1$. [See also example 2, section 11.5.]

4. To parametrize the ellipse $\dfrac{(x-h)^2}{a^2} + \dfrac{(y-k)^2}{b^2} = 1$, one can take $x(t) = h + a\cos t$, $y(t) = k + b\sin t$.

Example 4: Parametrize the circle $(x-1)^2 + (y+2)^2 = 9$.

Take $x(t) = 1 + 3\cos t$, $y(t) = -2 + 3\sin t$, $0 \le t \le 2\pi$.

5. To reverse the orientation of a given parametrization, substitute $-t$ for t.

Example 5: Reverse the orientation of the curve parametrized in Example 1, so that the curve goes from (4, 2) to (1, 1).

Let $x(t) = (-t)^2 = t^2$, $y(t) = -t$, and let $1 \le -t \le 2$, that is $-2 \le t \le -1$.

6. To shift the parameter of a given parametric curve by h units, substitute $t - h$ for t.

Example 6: Shift the parameter in Example 5 so that $0 \le t \le 1$.

We want to shift the parameter 2 units in the positive direction, so substitute $t - 2$ for t. Let $x(t) = (t-2)^2$, $y(t) = -(t-2)$, and $-2 \le t - 2 \le -1$, that is, $0 \le t \le 1$. To check this parameterization, one can verify that $x(0) = 4, y(0) = 2, x(1) = 1, y(1) = 1$.