Theorems Generalizing the Fundamental Theorem of Calculus

When faced with a line integral $\int_C \mathbf{F} \cdot d\mathbf{r}$, one or more theorems may be applicable, depending on whether C is a closed curve, that is, a curve forming the boundary a plane region R or a surface S; or whether the integrand is "exact," that is, a gradient. The following table summarizes when the various theorems can be applied.

To find $\int_C \mathbf{F} \cdot d\mathbf{r}$	$\mathbf{F}$ not conservative	$\mathbf{F} = \nabla f$ for some f
C not a closed curve	Calculate directly	Fund. Thm. of line integrals
C a closed curve	Green's (plane) Stokes' (surface)	0

When faced with a surface integral $\int_S\int \mathbf{F} \cdot \mathbf{n}\, dS$, one or more theorems may be applicable, depending on whether S is a closed surface, that is, a surface forming the boundary a solid region E; or whether the integrand is "exact," that is, a curl. The following table summarizes when the various theorems can be applied.

To find $\int_S\int \mathbf{F} \cdot \mathbf{n}\, dS$	$\mathbf{F} \neq \mathbf{curl}\,\mathbf{G}$ for some $\mathbf{G}$	$\mathbf{F} = \mathbf{curl}\,\mathbf{G}$ for some $\mathbf{G}$
S is not a closed surface	Calculate directly	Stokes' Theorem
S is boundary of solid E	Divergence Theorem	0