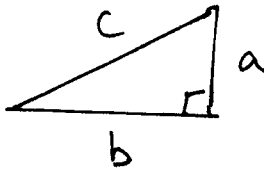
2.1 and 2.2: Rectangular coordinates, distance formula, equations of circles

Pythagorean theorem

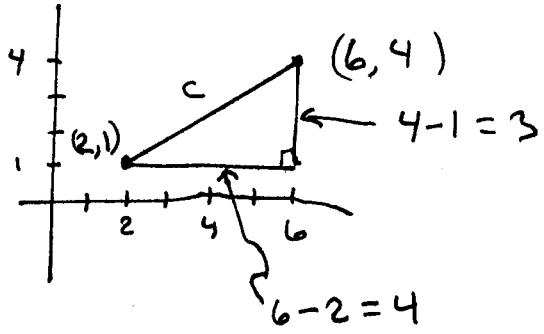


$$a^2+b^2=c^2$$

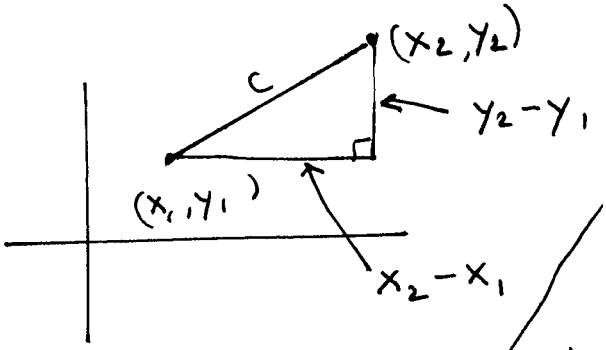
ex: $c = 13$ ft, $b = 12$ ft
what is a?

$$a^2 + 12^2 = 13^2$$
$$a^2 + 144 = 169$$
$$a^2 = 169-144 = 25$$
$$a = \sqrt{25} = 5 \text{ ft}$$

ex: What is the distance between $(2,1)$ and $(6,4)$?



$$c^2 = (6-2)^2 + (4-1)^2$$
$$c = \sqrt{(6-2)^2 + (4-1)^2}$$
$$= \sqrt{4^2+3^2} = \sqrt{16+9}$$
$$= \sqrt{25} = 5$$



$$c^2 = (x_2-x_1)^2 + (y_2-y_1)^2$$

$$c = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$$

= distance between (x_1, y_1) and (x_2, y_2)

ex [2.1 #18] a) Find the distance between $P(-8,4)$, $Q(3,-5)$

$$d = \sqrt{[3-(-8)]^2 + (-5-4)^2}$$

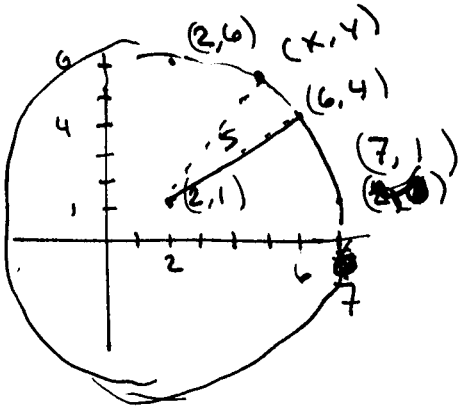
$$= \sqrt{11^2 + (-9)^2} = \sqrt{121+81}$$

$$= \sqrt{202} \approx 14.21$$

b) midpoint? $\left(\frac{-8+3}{2}, \frac{4+(-5)}{2}\right)$

$$= \left(\frac{-5}{2}, \frac{-1}{2}\right) = (-2.5, -0.5)$$

ex: What can we say about the coordinates of a point which lies 5 units from (2,1)?



$$5 = d = \sqrt{(x-2)^2 + (y-1)^2}$$

$$5^2 = (x-2)^2 + (y-1)^2$$

Answer: the coordinates (x,y) satisfy this equation.

Equation of a circle of radius 5, and center at (2,1)

$$\boxed{(x-2)^2 + (y-1)^2 = 25}$$

more generally "center-radius form" of the equation of a circle

$$\boxed{(x-h)^2 + (y-k)^2 = r^2}$$

where (h,k) = center r = radius

examples:

Equation	Center	Radius
$(x-3)^2+(y-1)^2=16$	$(3,1)$	4
$(x+2)^2+y^2=49$	$(-2,0)$	7
$(x+3)^2+(y-1)^2=8$	$(-3,1)$	$\sqrt{8}=2\sqrt{2}$
$x^2+y^2=1$	$(0,0)$	1

example:

Center	Radius	Equation
$(3,5)$	10	$(x-3)^2+(y-5)^2=100$
$(\frac{1}{2},\frac{3}{2})$	$\frac{1}{2}$	$(x-\frac{1}{2})^2+(y-\frac{3}{2})^2=\frac{1}{4}$

Remark: What if the equation is NOT in center-radius form.

example: $(x-2)^2+(y-1)^2=25$

$$x^2-4x+4+y^2-2y+1=25$$

$$x^2+y^2-4x-2y-20=0$$

General form of a circle →

$$\boxed{x^2+y^2+Dx+Ey+F=0}$$

How do we go back?

$$x^2+y^2-4x-2y-20=0$$

← what are the center and radius of this circle?

$$x^2-4x \qquad y^2-2y \qquad = 20$$

$$x^2-4x+4 \quad +y^2-2y+1 \quad = 20+4+1$$

$$(x-2)^2 + (y-1)^2 = 25$$

Aha! The center is $(2,1)$
radius is 5.

"complete the square":

i) Half of -4: $\frac{-4}{2}=-2$

ii) Square -2: $(-2)^2=4$

So 4 is the number which completes the square:

$x^2-4x+4=(x-2)^2$

i) $\frac{-2}{2}=-1$

ii) $(-1)^2=1$

So 1 is the number which completes the square:

$y^2-2y+1=(y-1)^2$