2.4 Linear functions

How we name polynomial functions

Degree	Name	Example
0	constant	$f(x) = 5$
1	linear	$f(x) = 2x - 3$
2	quadratic	$f(x) = x^2 - 8x + 15$
3	cubic	$f(x) = x^3 - 4x$
4	quartic	$f(x) = -x^4 + 2x^2 - 1$

A linear function:

$$\boxed{f(x) = ax + b}$$

where a and b are real numbers.
[Slope - intercept form]

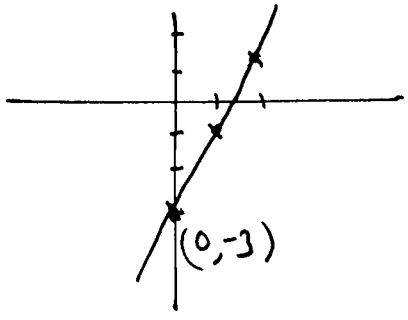
ex: $f(x) = 2x - 3$

So $f(10) = 2(10) - 3 = 17$

Domain = all reals

Range = all reals

x	y
0	-3
1	-1
2	1
3	3
4	5



Standard form of a line

$$Ax + By = C$$

ex: $2x + 3y = 12$

Note: The equation is not unique for a given line.

e.g., $20x + 30y = 120$ represents the same line.

what is the slope of this line?

$$2x + 3y = 12$$

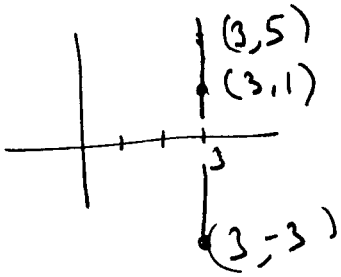
$$3y = -2x + 12$$

$$y = -\frac{2}{3}x + 4 \qquad \text{slope} = -\frac{2}{3}$$

Remark: why not always use slope-intercept form?

Answer: Slope intercept form can't handle vertical line (so undefined slope.)

ex: $x = 3$ OR $1x + 0y = 3$



← Is this the graph of y as a function of x?

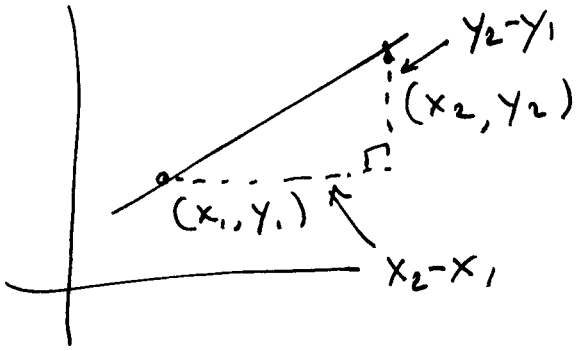
In fact, if a line is the graph of a function, it can be handled by slope-intercept form.

Remarks (1) If $B=0$ the line is vertical.

(2) If $A=0$ the line is horizontal.

(3) If $C=0$ the line passes through $(0,0)$.

Slope formula



$$\text{slope} = \frac{\text{rise}}{\text{run}} = \frac{y_2-y_1}{x_2-x_1}$$

ex:

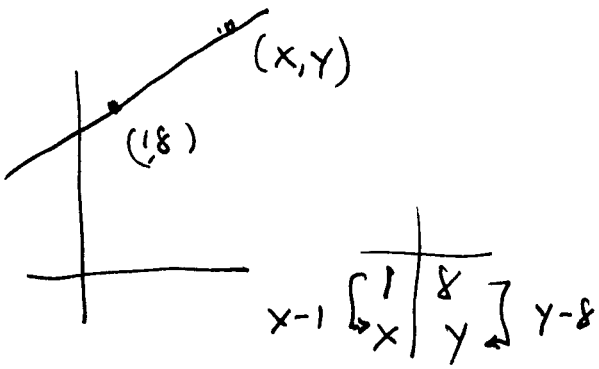
x	y
-3	5
1	8

(4 rows; 3 columns) ← A linear function.

What is the slope of the line

$$\text{slope} = \frac{y_2-y_1}{x_2-x_1} = \frac{8-5}{1-(-3)} = \frac{3}{4}$$

ex: Suppose the slope of line is $\frac{3}{4}$ and $(1,8)$ is a point on the line. What can say about (x,y) if (x,y) is another point on the line?



$$\frac{3}{4} = \frac{y-8}{x-1} \quad \text{so}$$

$$y-8 = \frac{3}{4}(x-1)$$

$$\boxed{y-y_1 = m(x-x_1)}$$

point-slope form of a line [§2.5].