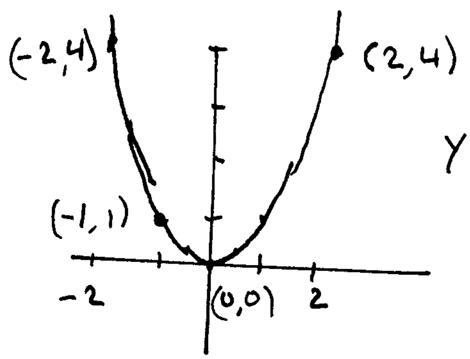
3.1 Quadratic functions

ex: $f(x) = x^2$

x	x^2
-2	4
-1	1
0	0
1	1
2	4
3	9

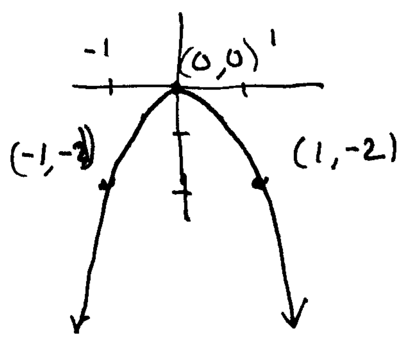


$y = x^2$

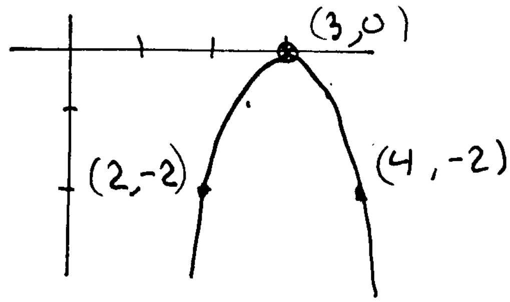
ex: $g(x) = -2(x-3)^2 + 2$

Think in terms of transformation of graphs

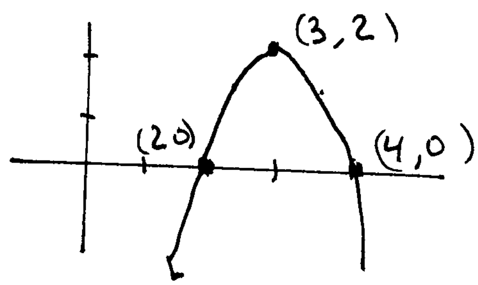
Step 1 $y = -2x^2$



Step 2 $y = -2(x-3)^2$



Step 3 $g(x) = -2(x-3)^2 + 2$



$$\boxed{f(x) = a(x-h)^2 + k}$$

"Vertex form" (or "standard form") of the equation of a quadratic function

where

(h,k) = vertex

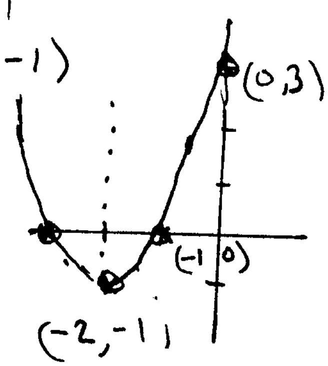
the parabola is open <u>up</u> if $a > 0$.

<u>ex</u>: $f(x) = (x+2)^2 - 1$ so $a = 1$, $h = -2$, $k = -1$

[eqn → graph]

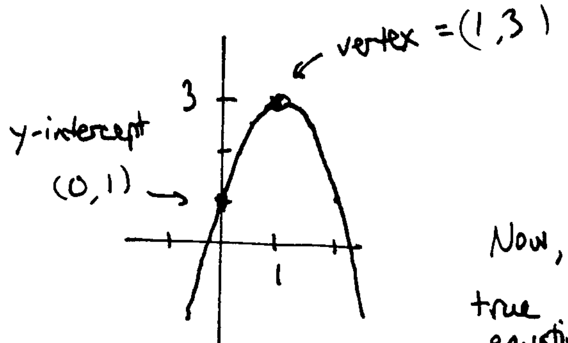
open up

vertex = $(-2,-1)$



x	y
-4	3
-3	0
-2	-1
-1	0
0	3

<u>ex</u>: Here is the <u>graph</u> of a quadratic function. Find its equation.



$(h,k) = (1,3)$

so the equation has the form

$$f(x) = a(x-1)^2 + 3$$

All we need is to determine a.

Now, use that $(x,y) = (0,1)$ satisfies the equation

true equation → $1 = a(0-1)^2 + 3$

so $1 = a + 3 \Rightarrow a = -2$

Answer: $\boxed{f(x) = -2(x-1)^2 + 3}$

$$\boxed{f(x) = ax^2 + bx + c}$$

"General form" of a quadratic function

Fact: A quadratic function in GENERAL form can always be written in VERTEX form.

ex.: $f(x) = 2x^2 + 12x + 17$

(The hard way)

$$= 2(x^2 + 6x \quad) + 17$$
$$= 2(x^2 + 6x + 9) + 17 - 18$$
$$= 2(x+3)^2 - 1$$

(The easy way) The easy to convert from general to vertex form is:

$$\boxed{\begin{aligned} a &= a \\ h &= -\frac{b}{2a} \\ k &= f(h) \end{aligned}}$$

How to remember:

$$\frac{\boxed{-b} \pm \sqrt{b^2 - 4ac}}{\boxed{2a}}$$

Also, to convert from vertex to general form, just multiply out.

ex: 34) $f(x) = -3x^2 + 24x - 46$ Convert to vertex form:

(easy way)

$$h = -\frac{b}{2a} = -\frac{24}{2(-3)} = \frac{24}{6} = 4$$

$$k = f(4) = -3 \cdot 4^2 + 24 \cdot 4 - 46 = -48 + 96 - 46 = 2$$

Answer: $f(x) = -3(x-4)^2 + 2$

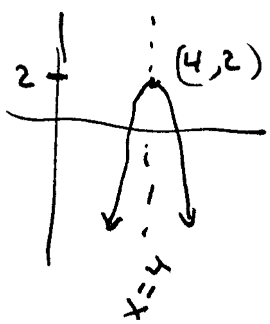
34 contd) a) Find the vertex:

vertex = (4, 2)

b) Axis of symmetry? The line $x=4$.

c) Domain of f? All reals = $(-\infty, \infty)$

d) Range of f? $(-\infty, 2]$



e) where is the function increasing? $(-\infty, 4)$

f) where is f decreasing? $(4, \infty)$

3.2 Synthetic Division

<u>Horner's Algorithm</u> (for evaluating polynomials)

<u>ex</u>: For the polynomial function

$$f(x) = 3x^3 + x^2 + 4x + 2$$

Evaluate f(2). But wait!!

$$\begin{aligned} f(x) &= (3x^3 + x^2 + 4x) + 2 \\ &= (3x^2 + x + 4)x + 2 \\ &= [(3x^2 + x) + 4]x + 2 \\ &= [(3x+1)x + 4]x + 2 \end{aligned}$$

<u>Note</u>: The coefficients are still visible, still in the same order.

$$\begin{aligned} f(2) &= \left[(3\cdot 2+1)\cdot 2+4\right]\cdot 2+2 \\ &= (7\cdot 2+4)\cdot 2+2 \\ &= 18\cdot 2+2 \\ &= 38 \end{aligned}$$

Bookeeping:

$$\begin{array}{r|rrrr} 2 & 3 & 1 & 4 & 2 \\ & & 6 & 14 & 36 \\ \hline & 3 & 7 & 18 & 38 \end{array} = f(2)$$

$$\begin{array}{r|rrrr} -3 & 3 & 1 & 4 & 2 \\ & & -9 & 24 & -84 \\ \hline & 3 & -8 & 28 & -82 \end{array} = f(-3)$$

We will see that these other numbers have meaning as well.