3.2 Synthetic (and Long) Division (cont'd)

__Review__ of long division of integers: regress to 3rd grade

We have 47 wooden blocks. How many stacks of 3 blocks can we make? How many blocks are left over?

$$47 = 3 + 44 = 3 + 3 + 41 = 3+3+3+38$$
$$= 10 \cdot 3 + 17$$
$$= 10 \cdot 3 + 5 \cdot 3 + 2$$
$$= 15 \cdot 3 + 2$$

$$
\begin{array}{rrl}
 & 15 & \leftarrow \text{quotient} \\
\text{divisor} \rightarrow 3 & \overline{)\,47} & \leftarrow \text{dividend} \\
 & \underline{30} & \\
 & 17 & \\
 & \underline{15} & \\
 & 2 & \leftarrow \text{remainder}
\end{array}
$$

$$47 = 3 \cdot 15 + 2$$

OR

$$\frac{47}{3} = 15 + \frac{2}{3}$$

$$
\begin{array}{rl}
 & x^2 + 5x - 15 \quad \leftarrow \text{quotient} \\
\text{divisor} \rightarrow x^2+3x+2 & \overline{)\,x^4 + 8x^3 + 2x^2 + 5x + 1} \quad \leftarrow \text{dividend} \\
 & \underline{x^4 + 3x^3 + 2x^2} \\
 & \quad 5x^3 + 0x^2 + 5x + 1 \\
 & \quad \underline{5x^3 + 15x^2 + 10x} \\
 & \qquad -15x^2 - 5x + 1 \\
 & \qquad \underline{-15x^2 - 45x - 30} \\
 & \qquad\qquad 40x + 31 \quad \leftarrow \text{remainder}
\end{array}
$$

This says:

$$x^4+8x^3+2x^2+5x+1 = (x^2+3x+2)\cdot(x^2+5x-15) + (40x+31)$$

OR

$$\frac{x^4+8x^3+2x^2+5x+1}{x^2+3x+2} = x^2+5x-15 + \frac{40x+31}{x^2+3x+2}$$

ex: Do long division to calculate, for $f(x) = 3x^3+x^2+4x+2$

$$f(x) \div (x-2) = \frac{3x^3+x^2+4x+2}{x-2}$$

$$\begin{array}{r}
3x^2 + 7x + 18 \\
x-2 \overline{)\, 3x^3 + x^2 + 4x + 2} \\
\underline{3x^3 - 6x^2 } \\
7x^2 + 4x + 2 \\
\underline{7x^2 - 14x } \\
18x + 2 \\
\underline{18x - 36} \\
38
\end{array}$$

so $f(x) = (x-2)\cdot(3x^2+7x+18) + 38$

Compare with

$$\begin{array}{c|cccc}
2 & 3 & 1 & 4 & 2 \\
 & & 6 & 14 & 36 \\
\hline
 & 3 & 7 & 18 & 38
\end{array}$$

$\underbrace{3 \quad 7 \quad 18}_{\text{quotient coefficients}}$

$38 = f(2)$ by Horner's algorithm
= remainder due to dividing by $(x-2)$

Behold: $f(2) = (2-2)\cdot(3\cdot 2^2 + 7\cdot 2 + 18) + 38$
$= 0 \cdot (\text{whatever}) + 38 = 38$

Remainder Theorem: If the polynomial $f(x)$ is divided by $x-k$, then the remainder is equal to $f(k)$.

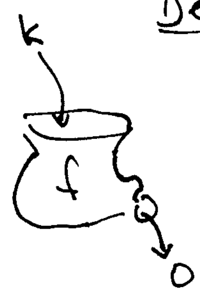
3.2 Use {Horner's algorithm / synthetic division} to perform:

12)
$$\frac{x^4+5x^3+4x^2-3x+9}{x+3} = x^3+2x^2-2x+3$$

$$\begin{array}{r|rrrr|r}
-3 & 1 & 5 & 4 & -3 & 9 \\
 & & -3 & -6 & 6 & -9 \\
\hline
 & 1 & 2 & -2 & 3 & 0
\end{array} = f(-3)$$

so NO remainder

OR $x^4+5x^3+4x^2-3x+9 = (x+3)(x^3+2x^2-2x+3)$



Definition: We say that k is a zero of a function $f(x)$ if $f(k)=0$.

example: In the previous example, we just showed that -3 is a zero of $f(x) =$ 4th degree polyn. function

The four following ideas are equivalent, for a polynomial function f

1. k is a zero of f.
2. k is a solution of the equation $f(x)=0$.
3. $(x-k)$ is a factor of $f(x)$.
4. The point $(k, 0)$ is an x-intercept of the graph $y=f(x)$.

3.3 Factor theorem: For any polynomial function $f(x)$, $x-k$ is a factor of $f(x)$ if and only if $f(k)=0$.

Reason: This is a special case of the Remainder Theorem when the remainder is 0.