# 3.3 Zeros of polynomial functions

§3.2 #65-72) $f(x) = x^3 - 2x^2 - x + 2$

Use synthetic division to evaluate:

row 1 $f(-2) = -12$

row 2 $f(-1) = 0$

row 3 $f\left(-\frac{1}{2}\right) = \frac{15}{8}$

row 4 $f(1) = 0$

row 5 $f\left(\frac{3}{2}\right) = -\frac{5}{8}$

row 6 $f(2) = 0$

row 7 $f(3) = 8$

row's work:

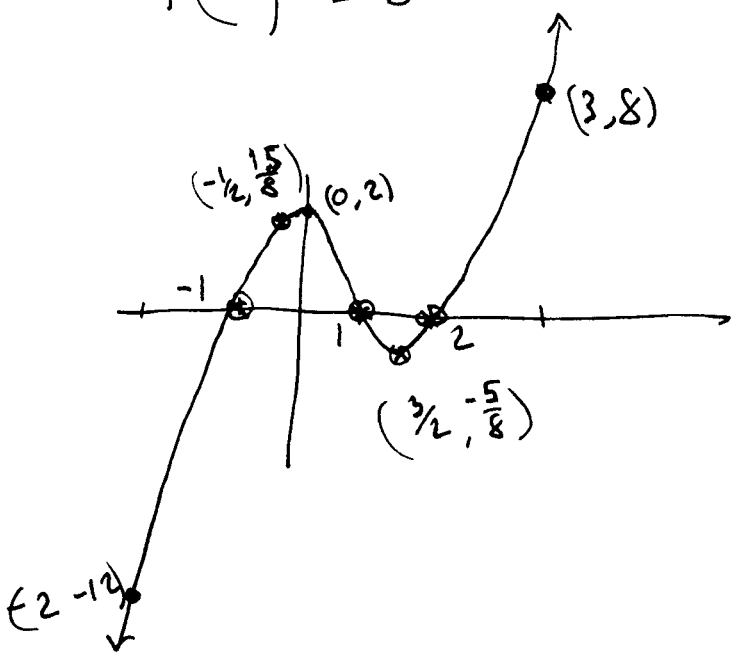
$$\begin{array}{r|cccc} 2 & 1 & -2 & -1 & 2 \\ & & 2 & 0 & -2 \\ \hline & 1 & 0 & -1 & 0 = f(2) \end{array}$$

coefficients of the quotient

Aha! $f(x) = (x-2)(x^2-1)$

$= (x-2)(x-1)(x+1)$

zeros: $2, 1, -1$



## Rational zeros theorem

If $\frac{p}{q}$ is a rational number, and is a zero of a polynomial function f with integer coefficients, then p is a factor of the constant term and q is a factor of the leading coefficient.

ex: $f(x) = 6x^3 + 19x^2 + 2x - 3$

a) List all possible rational zeros.

p: $\pm 1, \pm 3$

q: $\pm 1, \pm 2, \pm 3, \pm 6$

$\frac{p}{q}$: $\pm 1, \pm 3, \pm \frac{1}{2}, \pm \frac{3}{2}, \pm \frac{1}{3} \pm \frac{1}{6}$

b) Find the actual zeros, [Cheat by using a graphing calculator.]

By the graph it looks like $(-3, 0)$ is a x-intercept.

Confirm that:

$$\begin{array}{r|cccc} -3 & 6 & 19 & 2 & -3 \\ & & -18 & -3 & 3 \\ \hline & 6 & 1 & -1 & 0 = f(-3) \end{array}$$

$$f(x) = (x+3)(6x^2 + x - 1)$$
$$= (x+3)(2x+1)(3x-1)$$

zeros: $-3, -\frac{1}{2}, \frac{1}{3}$

## Number of zeros

A polynomial function of degree $n$ has $n$ zeros (counting multiplicity). Some may be complex numbers.

ex: $f(x) = x^3 + 3x^2 + 3x + 1$

$= (x+1)(x+1)(x+1) = (x+1)^3$

zeros: $-1, -1, -1$

We say "$-1$ is a zero of $f(x)$ of multiplicity 3."

58) Find a cubic polynomial function $f(x)$ with these conditions:

2 is a zero,

4 is a zero of multiplicity 2,

$f(1) = -18$.

$f(x) = a(x-2)(x-4)^2$

$-18 = f(1) = a(1-2)(1-4)^2$

$-18 = a(-1)(-3)^2 = -9a \Rightarrow a = \frac{-18}{-9} = 2$

$f(x) = 2(x-2)(x-4)^2 = 2(x-2)(x^2-8x+16)$

$= 2(x^3 - 10x^2 + 32x - 32)$

$= 2x^3 - 20x^2 + 64x - 64$

Scratch work:

$$\begin{array}{r} x^2-8x+16 \\ \hline x-2 \\ \hline -2x^2+16x-32 \\ x^3-8x^2+16x \\ \hline x^3-10x^2+32x-32 \end{array}$$