Loose end in §3.3 Descartes' Rule of Signs

§3.3 #84) $f(x) = 3x^4 + 2x^3 - 8x^2 - 10x - 1$

1 change in sign $\Rightarrow$ There will 1 positive zero.

$f(-x) = 3x^4 - 2x^3 - 8x^2 + 10x - 1$

3 changes in sign $\Rightarrow$ 3 negative zeros OR 1 negative zero

What's possible

positive	negative	nonreal complex zeros
1	3	0
1	1	2

Why 2? Because...

Conjugate zeros theorem: Nonreal zeros appear as conjugate pairs, (provided the polynomial function has real coefficients).

ex: $f(x) = x^2 - 4x + 5$

zeros? Solve $x^2 - 4x + 5 = 0$

$x^2 - 4x + 4 = -5 + 4$

$(x-2)^2 = -1$

$x - 2 = \pm\sqrt{-1} = \pm i$

$x = 2 \pm i$

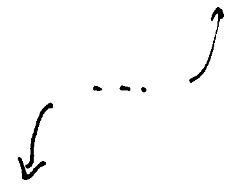
So $2+i$ and $2-i$ are (conjugate) zeros.

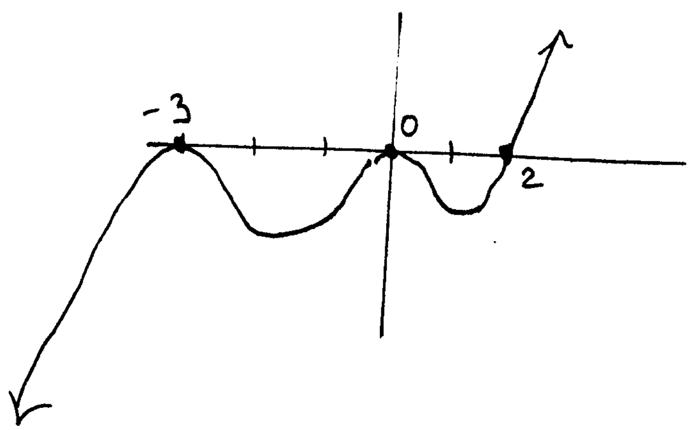
§3.4 Graphs of polynomial functions

33) $f(x) = x^2(x-2)(x+3)^2$ degree $= 5$

zeros: $0, 0, 2, -3, -3$

end behavior:





30) $f(x) = x^3 + 3x^2 - 13x - 15$

Number of zeros: 3

" of positive zeros: 1 change in sign $\Rightarrow$ 1 positive zero.

" " negative zeros: 2 changes $\Rightarrow$ 2 or 0 negative zeros.

because $f(-x) = -x^3 + 3x^2 + 13x - 15$

Rational zeros: $\pm 1, 3, 5, 15$ Is 3 a zero?

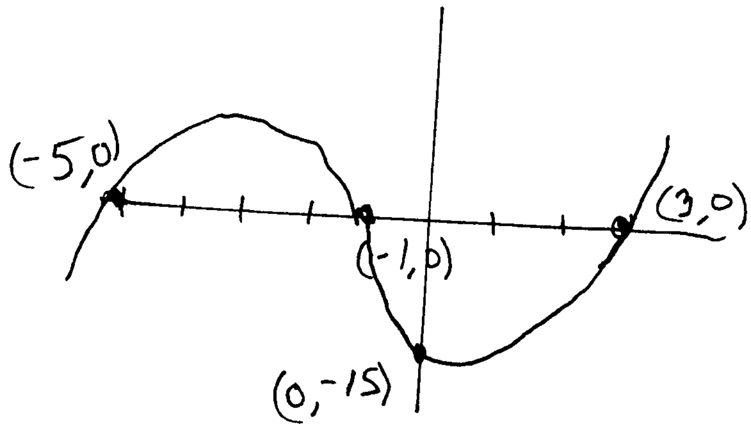
$$\begin{array}{r|rrrr} 3 & 1 & 3 & -13 & -15 \\ & & 3 & 18 & 15 \\ \hline & 1 & 6 & 5 & 0 \end{array}$$

$f(x) = (x-3)(x^2+6x+5)$

$= (x-3)(x+5)(x+1)$

zeros $3, -5, -1$

§3.4
30)
cont'd



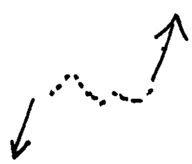
See the textbook (green boxes mostly) for:

$\boxed{\text{END BEHAVIOR}}$

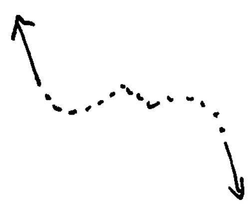
Odd degree

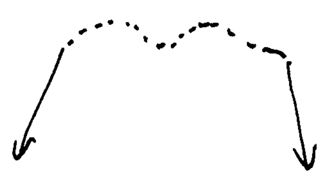
Even degree

Positive Leading coefficient



Negative leading coefficient

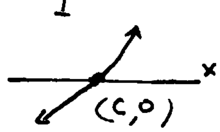




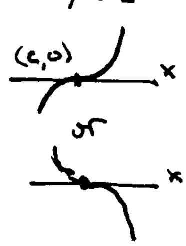
$\boxed{\text{BEHAVIOR AT ZEROS}}$ If c is a zero, then (c,0) will be an x-intercept. The behavior near (c,0) will depend on the multiplicity of the zero.

multiplicity:

1



odd, >1



even