Math 116 Wed 19 Oct 2016

3.5 Rational functions

Find Asymptotes for $f(x) = \frac{p(x)}{q(x)}$

1. vertical asymptotes: Occur at the zeros of $q(x)$.

2. Other asymptotes:

$f(x) = \frac{p(x)}{q(x)}$ is a proper rational function →

a) If degree of $p(x) <$ degree of $q(x)$ then the line $y=0$ is a horizontal asymptote.

$f(x) = \frac{p(x)}{q(x)}$ is an improper rational function. →

b) If degree of $p(x) =$ degree of $q(x)$ then $y = \frac{a}{b}$ is a horizontal asymptote, where $a =$ leading coefficient of $p(x)$, and $b =$ leading coefficient of $q(x)$.

c) If degree of $p(x) = 1 +$ degree of $q(x)$ then there is an oblique (or slant) asymptote. We get its equation by doing long division.

examples	vertical asymptotes	other asymptotes
$f(x) = \frac{x-3}{(x+1)(x-2)}$	$x=-1$, $x=2$	Horizontal: $y=0$
$f(x) = \frac{(3x+1)(x-1)}{(2x-1)(x+3)}$	$x=-3$, $x=\frac{1}{2}$	Horizontal: $y=\frac{3}{2}$
$f(x) = \frac{x^2-2x+2}{x-1}$	$x=1$	Slant: $y=x-1$

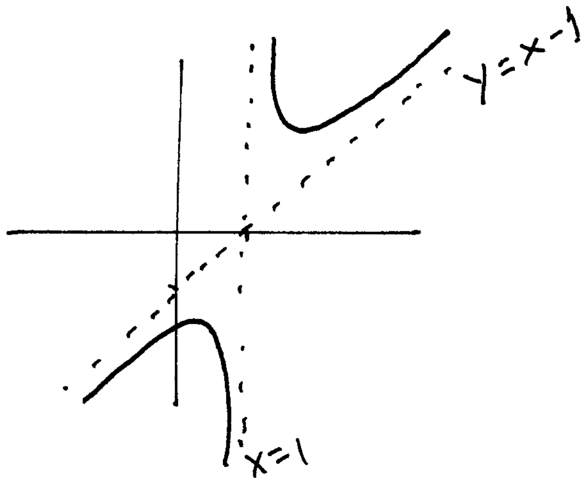
ex: For $f(x) = \dfrac{x^2-2x+2}{x-1}$ what is the slant asymptote?

$$\begin{array}{r} x-1 \\ x-1 \overline{) x^2-2x+2} \\ \underline{x^2-x} \\ -x+2 \\ \underline{-x+1} \\ 1 \end{array}$$

OR

$$\begin{array}{c|ccc} 1 & 1 & -2 & 2 \\ & & 1 & -1 \\ \hline & 1 & -1 & 1 \end{array}$$

So $f(x) = \dfrac{x^2-2x+2}{x-1} = x-1+\dfrac{1}{x-1}$



68) $f(x) = \dfrac{2x+1}{x^2+6x+8} = \dfrac{2x+1}{(x+4)(x+2)}$

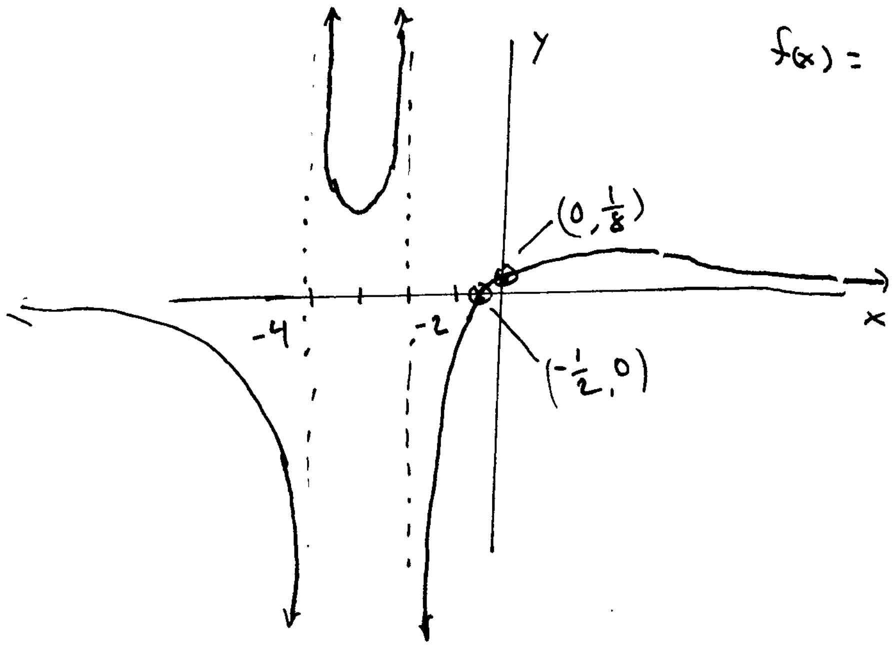
① zeros of bottom? -4 and -2 so $x=-4$ and $x=-2$ are vertical asymptotes.

② zeros of top? $-\frac{1}{2}$ So $(-\frac{1}{2},0)$ is the x-intercept.

③ horizontal asymptote? Yes, $y=0$ because $f(x)$ is a proper rational function.

④ $f(0) = \frac{1}{8}$ so $(0,\frac{1}{8}) = y$-intercept

$$f(x) = \frac{2x+1}{(x+4)(x+2)}$$



<u>Remark</u>: The only locations where the y-values can <u>change</u> sign are at the x-intercepts (where $p(x)=0$) or at the vertical asymptotes (where $q(x)=0$).

Moreover, the y-values <u>will</u> change sign only if the respective zeros have odd multiplicity.

<u>In the above example</u>, the graph <u>changed</u> sign at $x = -\frac{1}{2}$ (a zero of multiplicity one of the numerator) and at each of $x = -2$ and $x = -4$, (each zeros of multiplicity one of the denominator).

§3.5 #68) $f(x) = \dfrac{2x+1}{x^2+6x+8} = \dfrac{2x+1}{(x+4)(x+2)}$

a better sketch of the graph.