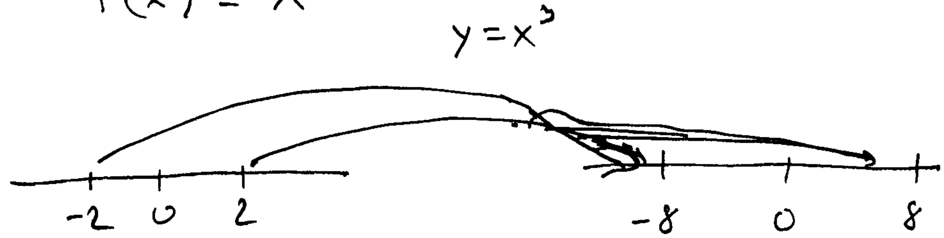
4.1 Inverse functions

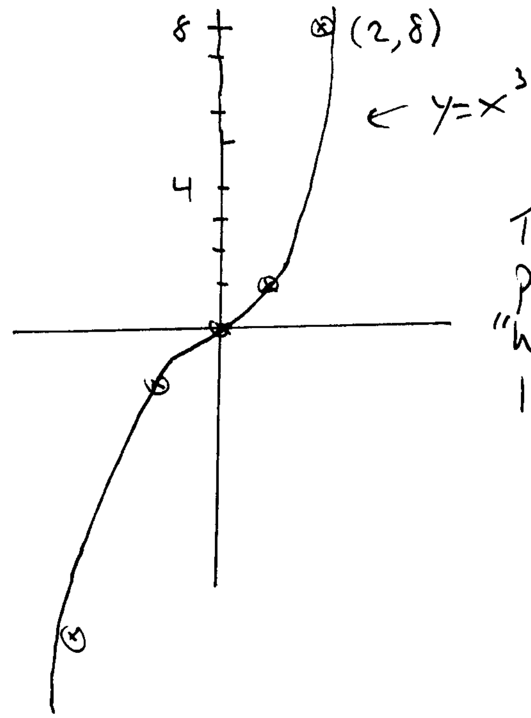
<u>Defn</u>: A function f is <u>one-to-one</u> if $a \neq b$ implies that $f(a) \neq f(b)$

<u>example</u>: $f(x) = x^3$



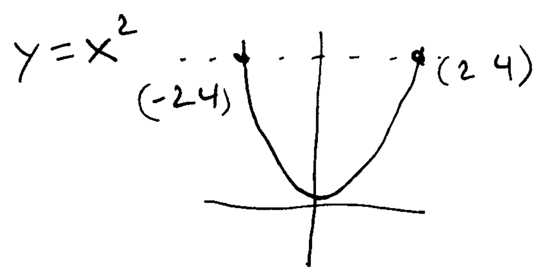
x	x^3
-3	-27
-2	-8
-1	-1
0	0
1	1
2	8
3	27
4	64

No repeats in 2nd column



The graph passes the "horizontal line test".

<u>example</u> of a function which is <u>NOT</u> one-to-one.



x	x^2
-3	9
-2	4
-1	1
0	0
1	1
2	4
3	9

Defn: Let f be a one-to-one function.
Then g is the inverse function of f if:

(1) $(f \circ g)(x) = f(g(x)) = x$ for every x in the domain of g.

and (2) $(g \circ f)(x) = g(f(x)) = x$ for every x in the domain of f.

ex: 42) $f(x) = 3x + 9$, $g(x) = \frac{1}{3}x - 3$

Are f and g inverses of each other.

$$f(g(x)) = f\left(\tfrac{1}{3}x - 3\right) = 3\left(\tfrac{1}{3}x - 3\right) + 9$$
$$= x - 9 + 9 = x$$

Also
$$g(f(x)) = g(3x+9) = \tfrac{1}{3}(3x+9) - 3$$
$$= x + 3 - 3 = x$$

How to find the Inverse of $y = f(x)$, where f is one-to-one

Step 1: Swap x and y.

Step 2: Solve for y.

Step 3: Replace y with $f^{-1}(x)$ ← Bad notation but we're stuck with it.

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64) $f(x) = -x^3 - 2$ (Believe f is one-to-one.)

Step 0: $y = -x^3 - 2$

Step 1: Swap: $x = -y^3 - 2$

Step 2: Solve for y.

$$x + 2 = -y^3 - 2 + 2$$

$$x + 2 = -y^3$$

$$-1(x+2) = -1 \cdot (-y^3)$$

$$-x - 2 = y^3$$

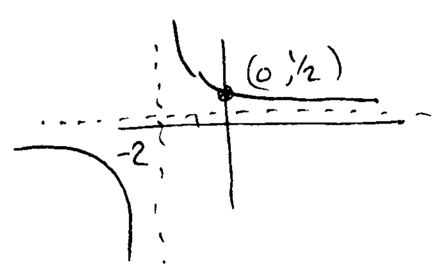
$$\sqrt[3]{-x-2} = \sqrt[3]{y^3}$$

$$\sqrt[3]{-x-2} = y$$

Step 3: $f^{-1}(x) = \sqrt[3]{-x-2}$

70) $f(x) = \dfrac{1}{x+2}$, $x \neq -2$. Is f one-to-one?

Graph $y = \dfrac{1}{x+2}$



Yes, its graph passes the horizontal line test.

Step 1: $x = \dfrac{1}{y+2}$, $y \neq -2$

Step 2:

Shortcut: Take reciprocal of both sides

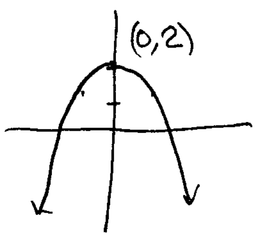
$$\frac{1}{x} = y + 2$$

$$\frac{1}{x} - 2 = y + 2 - 2$$

$$\frac{1}{x} - 2 = y$$

Step 3: $f^{-1}(x) = \dfrac{1}{x} - 2$

66) $f(x) = -x^2+2$ what gives if f is not one-to-one?



Fails the horizontal line test

Try to find f^{-1} anyway.

$$y = -x^2+2$$

Swap: $x = -y^2+2$

$$x-2 = -y^2+2-2$$

$$x-2 = -y^2$$

$$-(x-2) = -(-y^2)$$

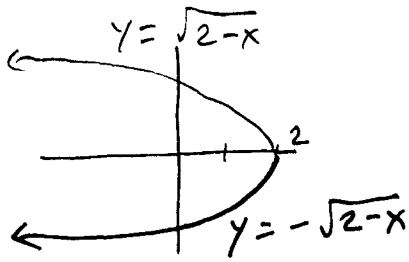
$$-x+2 = -(x-2) = y^2$$

$$y^2 = 2-x$$ Now what?

$$y = \pm\sqrt{2-x}$$ That is

$y = \sqrt{2-x}$ OR $y = -\sqrt{2-x}$

That is, we cannot uniquely solve for y.

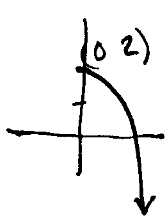


Remark: If we define

$$f(x) = \begin{cases} -x^2+2 & \text{if } x \geq 0 \\ \text{undefined} & \text{if } x < 0 \end{cases}$$

This is one-to-one and

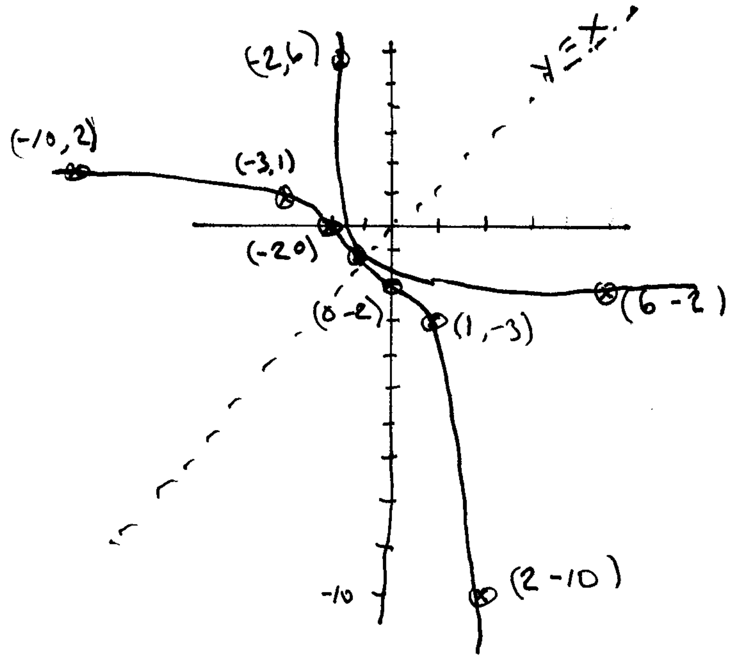
$$f^{-1}(x) = \sqrt{2-x}$$



back to 64) $f(x) = -x^3 - 2$

How to find the table or the graph of an inverse function.

x	$f(x)$
-2	6
-1	-1
0	-2
1	-3
2	-10



x	$f^{-1}(x)$
6	-2
-1	-1
-2	0
-3	1
-10	2