Math 116 Wed 2 Nov 2016

4.2 Exponential functions

Review of rational exponents

reciprocal 2nd power 3rd root

ex: $1000^{-2/3}$

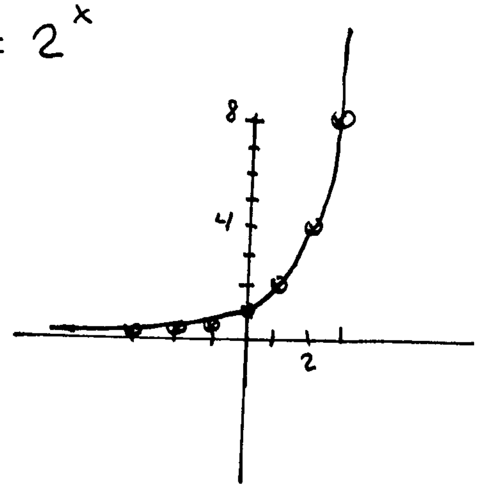
$$= \frac{1}{\left(\sqrt[3]{1000}\right)^2} = \frac{1}{10^2} = \frac{1}{100}$$

ex: $$\left(\frac{4}{25}\right)^{-1/2} = \left(\frac{25}{4}\right)^{1/2} = \sqrt{\frac{25}{4}} = \frac{5}{2}$$

example: [of an exponential function]

$$f(x) = 2^x$$

x	2^x
-2	$1/4$
-1	$1/2$
0	1
1	2
$1/2$	$\sqrt{2} \approx 1.4$
2	4
$3/2$	$2\sqrt{2} \approx 2.8$
3	8
4	16
5	32



Domain $= (-\infty, \infty)$

Range $= (0, \infty)$

Increasing on the domain

One-to-one

Horizontal asymptote: $y=0$

$f(\pi) = ?$

$f(\pi) \approx f\left(\frac{22}{7}\right) = \sqrt[7]{2^{22}}$

Exponential equations

ex: $2^x = 32$

$2^x = 2^5$ Because $f(x) = 2^x$ is one-to-one

if $2^x = 2^5$ then $x = 5$.

ex: Solve $10^x = \sqrt{10} \Rightarrow 10^x = 10^{1/2} \Rightarrow x = \frac{1}{2}$

ex: $16^x = 32$

$(2^4)^x = 2^5$

$2^{4x} = 2^5$

$4x = 5 \Rightarrow x = \frac{5}{4}$

ex: [4.2 #82] $2^{6-3x} = 8^{x+1}$

$2^{6-3x} = (2^3)^{x+1}$

$2^{6-3x} = 2^{3x+3}$

$6 - 3x = 3x + 3$

$+3x \quad +3x$

$6 = 6x + 3$

$-3 \quad -3$

$3 = 6x \Rightarrow \boxed{x = \frac{1}{2}}$

Compound interest

example: You initially have $P = \$1000$ in an account ~~with~~ which earns 6% annual interest, compounded quarterly.

1st quarter: Interest for a quarter year $\frac{6\%}{4} = 1.5\%$ per quarter

$$\text{interest} = 1.5\% \text{ of } \$1000$$
$$= .015 \cdot 1000 = \$15$$

New balance after 1st quarter $= 1000 + 1000 \cdot (.015)$
$$= 1000\,(1 + .015)$$
$$= 1000\,(1.015) = 1015 \text{ dollars}$$

Balance after 2nd quarter $= 1015 \cdot (1 + .015)$
$$= 1000\,(1 + .015)^2$$

⋮

Balance after t years, so after 4t quarters:

$$\text{Balance} = A = 1000\,(1 + .015)^{4t}$$
$$= 1000\left(1 + \frac{.06}{4}\right)^{4t}$$

$$\boxed{A = P\left(1 + \frac{r}{n}\right)^{nt}}$$

where P = principal = present value
A = amount after t years = future value
r = annual interest rate (per year)
n = number of compounding per year
t = time (years)
So nt = number of compoundings after t years

ex: If $P = \$1000$ $n=12$ $r = .06 = 6\%$
and $t = 3$ years

then $A = 1000\left(1 + \frac{.06}{12}\right)^{3 \cdot 12}$

$= 1000\,(1 + .005)^{36} = 1000\,(1.005)^{36}$

$= 1000\,(1.19668) = \boxed{1{,}196.68 \text{ dollars}}$

Continuous compounding

$$\boxed{A = Pe^{rt}}$$

where $e \approx 2.718281828$

Actually

$e = \lim_{n \to \infty} \left(1 + \frac{1}{n}\right)^n$

$\approx \left(1 + \frac{1}{\text{HUGE}}\right)^{\text{HUGE}}$

ex: If $P = \$1000$ $r = .06$ per yr and $t = 3$ yrs

$A = 1000\, e^{(.06)3} = 1000\, e^{.18}$

$= 1000\,(1.19722)$

$= \boxed{\$1{,}197.22}$