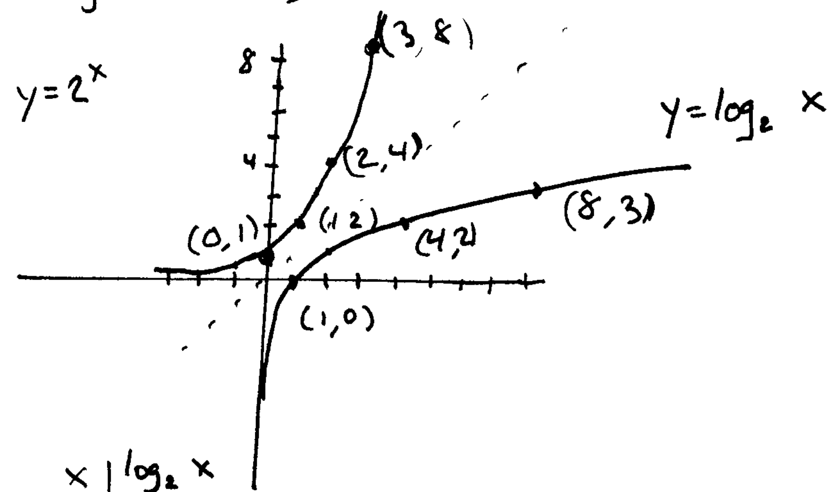
4.3 Log functions



x	2^x
-2	1/4
-1	1/2
0	1
1	2
2	4
3	8
4	16

x	$\log_2 x$
1/4	-2
1/2	-1
1	0
2	1
4	2
8	3
16	4

Defn: $y=\log_2 x$ means $2^y = x$

example: $3=\log_2 8$ means $2^3 = 8$

ex: $-2 = \log_2 \frac{1}{4}$ means $2^{-2} = \frac{1}{4}$

$\log_{\underset{\text{base}}{2}} \overset{\text{other}}{32} = \underbrace{5}_{\text{exponent}}$ means $\underset{\text{base}}{2}^{\overset{\text{exponent}}{5}} = \overset{\text{other}}{32}$

<u>Defn</u>:

$$\boxed{y = \log_a x \quad \text{is equivalent to} \quad a^y = x}$$

<u>Remark</u>: $f(x) = x^2$ f is <u>name</u> of the function

So $f(7) = 49$.

$\log_2 8 = \log_2(8) = 3$

parentheses are optional

$\log_2$ is the <u>name</u> of the function

ex [practice in equivalent equations]

$2^5 = 32$ is equivalent to $\log_2 32 = 5$

$10^3 = 1000$ " $\log_{10} 1000 = 3$

$10^6 = 1{,}000{,}000$ $\log_{10} 1{,}000{,}000 = 6$

$10^{-2} = 0.01$ $\log_{10} 0.01 = -2$

$10^{1/2} = \sqrt{10} \approx 3.16$ $\log_{10} \sqrt{10} = \frac{1}{2}$

$10^0 = 1$ $\log_{10} 1 = 0$

[other way] $\log_7 49 = 2$ is equivalent to $7^2 = 49$ are <u>both</u> true

$\log_7 \frac{1}{7} = -1$ " $7^{-1} = \frac{1}{7}$

ex: Find $\log_3 81$. Let $y = \log_3 81$.

Equivalent:

$$3^y = 81$$ Okay so what is y?

$$3^y = 3^4$$

$$y = 4$$

conclude: $\log_3 81 = 4$.

Shortcut: $\log_3 81 = \log_3 3^4 = 4$.

ex: Find (use shortcut: $\log_a a^x = x$)

$$\log_6 36 = \log_6 6^2 = 2$$

ex: $\log_2 512 = \log_2 2^9 = 9$

Log equations:

(1) [x in "base" position] $\log_x \frac{27}{64} = 3$

[#22] Exponential form: $x^3 = \frac{27}{64}$ so $x = \sqrt[3]{\frac{27}{64}} = \frac{3}{4}$

(2) [x in "other" position] $\log_5 x = 4$

Equivalent $5^4 = x$ so $x = 625$

(3) [x in "exponent" position] $\log_4 \sqrt[3]{16} = x$

[#33] Equivalent: $4^x = \sqrt[3]{16} = \sqrt[3]{4^2} = 4^{2/3}$

so $x = \frac{2}{3}$

4.3 (cont'd) Six Properties of log functions

(1) $\log_a a^x = x$

(2) $a^{\log_a x} = x$

inverse properties

(3) $\log_a xy = \log_a x + \log_a y$ product property

(4) $\log_a \frac{x}{y} = \log_a x - \log_a y$ quotient property

(5) $\log_a x^r = r \log_a x$ power property

(6) $\log_a x = \dfrac{\log_b x}{\log_b a}$ change-of-base theorem

examples: (1) $\log_3 3^5 = 5$, $\log_{10} 10^{1/2} = \frac{1}{2}$, $\log_7 7^0 = 0$.

$\log_{11} 11 = \log_{11} 11^1 = 1$, $\log_8 1 = \log_8 8^0 = 0$

(2) $2^{\log_2 8} = 8$, $10^{\log_{10} 3} = 3$, $5^{\log_5 625} = 625$

(3) $\log_{10} (100)(1000) = \log_{10} 100 + \log_{10} 1000$ That is,

$5 = 2 + 3$

(4) $\log_{10} \frac{100,000}{1000} = \log_{10} 100,000 - \log_{10} 1000$

$2 = 5 - 3$

Use properties of logs to rewrite the expressions:

72) $$\log_3 \frac{4p}{q} = \log_3 4p - \log_3 q$$

$$= \log_3 4 + \log_3 p - \log_3 q$$

78) $$\log_3 \sqrt[3]{\frac{m^5 n^4}{t^2}} = \log_3 \left(\frac{m^5 n^4}{t^2}\right)^{1/3}$$

$$= \frac{1}{3} \log_3 \left(\frac{m^5 n^4}{t^2}\right)$$ by ~~product~~ power rule

$$= \frac{1}{3} \left(\log_3 m^5 n^4 - \log_3 t^2\right)$$ by quotient rule

$$= \frac{1}{3} \left(\log_3 m^5 + \log_3 n^4 - \log_3 t^2\right)$$ product rule

$$= \frac{1}{3} \left(5 \log_3 m + 4 \log_3 n - 2 \log_3 t\right)$$ power rule

$$= \frac{5}{3} \log_3 m + \frac{4}{3} \log_3 n - \frac{2}{3} \log_3 t$$ algebra (optional)