4.4 Change-of-base theorem

ex: Find $\log_3 20$.

Let $y = \log_3 20$

$$3^y = 20$$

$$\log_{10} 3^y = \log_{10} 20$$ Now, use the power property.

$$y \log_{10} 3 = \log_{10} 20$$

$$\frac{y \cancel{\log_{10} 3}}{\cancel{\log_{10} 3}} = \frac{\log_{10} 20}{\log_{10} 3}$$

$$y = \frac{\log_{10} 20}{\log_{10} 3} = \frac{1.301}{0.477} = 2.727$$

mimic this example to show $\boxed{\log_a x = \frac{\log_b x}{\log_b a}}$

Replace 20 with x, 3 with a, and 10 with b.

Get: Change-of-base formula

In practice: $\boxed{\log_a x = \frac{\ln x}{\ln a}}$ or $\boxed{\log_a x = \frac{\log x}{\log a}}$

where $\ln x = \log_e x$ = "natural logarithm" and $\log x = \log_{10} x$ = "common logarithm"

Recall: $e \approx 2.718281828$

Evaluate to four decimal places.

12) $\log 10^7 = \log_{10} 10^7 = 7$

16) $\log 94 = 1.9731$

46) $\ln e^{5.8} = 5.8$

50) $\ln \sqrt[3]{e} = \log_e e^{1/3} = \frac{1}{3}$

52) $\ln 39 = 3.6636$

80) $\log_2 9 = \dfrac{\ln 9}{\ln 2} = \dfrac{2.1972}{0.6931} = 3.1699$

OR $\dfrac{\log 9}{\log 2} = \dfrac{0.9542}{0.3010} = 3.1699$

## 4.5 Exponential and Logarithmic Equations

ex: [a common base is possible]

$9^x = 27$

$(3^2)^x = 3^3$

$3^{2x} = 3^3$

$2x = 3$

$x = \frac{3}{2}$

Solve to three decimal places.

§4.5 12)

$$5^x = 13$$

$$\ln 5^x = \ln 13$$

$$x \ln 5 = \ln 13$$

exact answer

$$x = \frac{\ln 13}{\ln 5} \approx 1.594$$

← approximate answer

18)

$$2^{x+3} = 5^{2x}$$

$$\log 2^{x+3} = \log 5^{2x}$$

Now use the power property

$$(x+3)\log 2 = (2x)\log 5$$

$$(\log 2)x + 3\log 2 = (2\log 5)x$$

$$3\log 2 = (2\log 5)x - (\log 2)x$$

$$3\log 2 = (2\log 5 - \log 2)x$$

So

$$x = \frac{3\log 2}{2\log 5 - \log 2} = \frac{\log 2^3}{\log\left(\frac{5^2}{2}\right)} = \frac{\log 8}{\log 12.5}$$

$$\approx 0.823$$

§4.5 36) $e^{2x} - 8e^{x} + 15 = 0$

$$(e^x)^2 - 8e^x + 15 = 0$$

$$(e^x - 3)(e^x - 5) = 0$$

$e^x - 3 = 0$ OR $e^x - 5 = 0$

$e^x = 3$ $e^x = 5$

$x = \ln 3$ or $x = \ln 5$

$\approx 1.099$ $\approx 1.609$

Log Equations

ex: $\log_2(x-5) = 3$

$$x - 5 = 2^3$$
$$x - 5 = 8$$
$$x = 13$$

52) $\log x + \log x^2 = 3$ by the product property

$$\log(x \cdot x^2) = 3$$

$$\log x^3 = 3$$

$$x^3 = 10^3 = 1000$$

$$x = \sqrt[3]{1000} = 10$$

76) <u>Remark</u>: Our answer <u>must</u> be in domain of our log functions.

$$\log_2(x-7) + \log_2 x = 3$$

Domain? Must have $x-7>0$ so $x>7$

$$\log_2 (x-7)x = 3$$

$$\log_2 (x^2-7x) = 3$$

$$x^2 - 7x = 2^3$$

$$x^2 - 7x = 8$$

$$x^2 - 7x - 8 = 0$$

$$(x-8)(x+1) = 0$$

$$x-8=0 \quad \text{or} \quad x+1=0$$

$$\boxed{x=8} \quad \text{or} \quad \cancel{x=-1}$$ extraneous solution