WARM-UP

(1) Solve

$$16^{x+4} = 8^{3x-2}$$

$$(2^4)^{x+4} = (2^3)^{3x-2}$$

$$2^{4x+16} = 2^{9x-6}$$

$$4x+16 = 9x-6$$

$$-4x+6 \quad -4x+6$$

$$22 = 5x \Rightarrow x = \frac{22}{5} = \boxed{4.4}$$

(2) Solve

$$3^{2x-5} = 13$$

$$\ln 3^{2x-5} = \ln 13$$

$$(2x-5)\ln 3 = \ln 13$$

$$\frac{(2x-5)\ln 3}{\ln 3} = \frac{\ln 13}{\ln 3}$$

$$2x-5 = \frac{\ln 13}{\ln 3}$$

$$2x = 5 + \frac{\ln 13}{\ln 3}$$

"exact answer"

$$x = \frac{1}{2}\left(5 + \frac{\ln 13}{\ln 3}\right) = \frac{1}{2}\left(5 + \frac{2.5649}{1.0986}\right)$$

$$= \frac{1}{2}(5 + 2.3347)$$

$$= 3.6674 \approx \boxed{3.667}$$

M116 16.11.14

Warmup 3 $\log_3 (x^2-9) = 3$

Equivalent: $x^2 - 9 = 3^3 = 27$

$x^2 = 36$

$x = \pm\sqrt{36} = \boxed{\pm 6}$

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p 493 #74) How many years (to nearest 10th of year) are needed for \$48,000 to become \$53,647 at 2.8% interest, compounded semiannually.

$$A = P\left(1 + \frac{r}{n}\right)^{tn}$$

$$53{,}647 = 48{,}000\left(1 + \frac{.028}{2}\right)^{2t}$$

$$53{,}647 = 48{,}000 \cdot (1.014)^{2t}$$

$$1.117646 \approx \frac{53{,}647}{48{,}000} = (1.014)^{2t}$$

by the Power Rule

$$\log 1.117646 = \log 1.014^{2t} = 2t \cdot \log 1.014$$

$$= (2\log 1.014)\,t$$

$$t = \frac{\log 1.117646}{2\log 1.014} = 4.0 \text{ years}$$

4.5 Log equations (cont'd)

60) $\log(3x+5) - \log(2x+4) = 0$

$$\log(3x+5) = \log(2x+4)$$

$$3x+5 = 2x+4$$
$$-2x \quad -2x$$
$$x+5 = 4$$
$$-5 \quad -5$$
$$\boxed{x=-1}$$

and note that -1 is in domain of both log functions.

72) $\log(11x+9) = 3 + \log(x+3)$

$$\log(11x+9) - \log(x+3) = 3$$

Domain? Need:
① $x+3>0$ or $x>-3$, AND
② $11x+9>0$
$11x>-9$
$\boxed{x>-\frac{9}{11}}$

$$\log\left(\frac{11x+9}{x+3}\right) = 3$$

$$\frac{11x+9}{x+3} = 10^3 = 1000$$

$$11x+9 = 1000(x+3)$$
$$11x+9 = 1000x+3000$$
$$9-3000 = 1000x-11x$$
$$-2991 = 989x$$
$$x = -\frac{2991}{989} \approx -\frac{997}{33\cancel{7}} = -3.024$$

Not in the domain, so NO SOLUTION.

4.6 Applications of Exponential Growth + Decay

24) A sample from a refuse deposit had 60% of the carbon-14 of a contemporary sample. How old is it? Use

$$y = y_0 e^{kt}$$

where $k = -0.000\,1216$ per year

and $t =$ time (years)

So $y = 0.60\,y_0$.

Solve for t:

$$0.60\,\cancel{y_0} = \cancel{y_0}\, e^{-0.000\,1216t}$$

$$0.60 = e^{-0.000\,1216t}$$

$$\ln 0.60 = -0.000\,1216t$$

$$t = \frac{\ln 0.60}{-0.000\,1216} = 4{,}200 \text{ years old}$$

Remark: ① The interpretation of the decay rate $k = -0.000\,1216$ per year is that each year, a proportion of $0.000\,1216$ of the carbon-14 (or about 122 parts per million) disappears each year due to radioactive decay.

② Solve $\frac{1}{2} y_0 = y_0 e^{-0.000\,1216t}$ and we get $t = 5700$ years.
The interpretation of this number is that $5700 = \frac{\ln 2}{0.000\,1216}$ is the half-life of carbon-14.