5.2

Example 4 (p 521) Solve $\begin{cases} 2x - 5y + 3z = 1 \\ x - 2y - 2z = 8 \end{cases}$

$$\begin{bmatrix} 2 & -5 & 3 & 1 \\ 1 & -2 & -2 & 8 \end{bmatrix}$$

TI-84 commands:

$R_1 \leftrightarrow R_2$ $\begin{bmatrix} 1 & -2 & -2 & 8 \\ 2 & -5 & 3 & 1 \end{bmatrix}$ RowSwap(Ans, 1, 2)

$-2R_1 + R_2 \to$ $\begin{bmatrix} 1 & -2 & -2 & 8 \\ 0 & -1 & 7 & -15 \end{bmatrix}$ *row+(-2, Ans, 1, 2)

$-1R_2 \to$ $\begin{bmatrix} 1 & -2 & -2 & 8 \\ 0 & 1 & -7 & 15 \end{bmatrix}$ *row(-1, Ans, 2)

$2R_2 + R_1 \to$ $\begin{bmatrix} 1 & 0 & -16 & 38 \\ 0 & 1 & -7 & 15 \end{bmatrix}$ *row+(2, Ans, 2, 1)

"Reduced row-echelon form"

$\begin{cases} x \quad -16z = 38 \\ \quad y \quad -7z = 15 \end{cases}$

Note: we're free to let z be any number. Denote that number as "z".

Then $y = 7z + 15$ and $x = 16z + 38$

Solution set $= \{ (16z + 38, 7z + 15, z) \}$

ex: Write three of the solutions

$(x, y, z) = (38, 15, 0)$ or $(54, 22, 1)$ or $(22, 8, -1)$

Survey of topics we ran out of time for:

5.7 matrix properties

5.8 Matrix inverses

<u>Adding matrices and Scalar multiplication</u>

Two 2x4 matrices

$$A = \begin{bmatrix} 2 & 7 & 0 & -1 \\ 5 & -3 & 2 & 2 \end{bmatrix} \qquad B = \begin{bmatrix} -4 & -1 & 3 & 0 \\ 0 & 7 & 5 & 2 \end{bmatrix}$$

$$A + B = \begin{bmatrix} -2 & 6 & 3 & -1 \\ 5 & 4 & 7 & 4 \end{bmatrix}$$

$$10A = 10\begin{bmatrix} 2 & 7 & 0 & -1 \\ 5 & -3 & 2 & 2 \end{bmatrix} = \begin{bmatrix} 20 & 70 & 0 & -10 \\ 50 & -30 & 20 & 20 \end{bmatrix}$$

$$-\tfrac{1}{2}B = -\tfrac{1}{2}\begin{bmatrix} -4 & -1 & 3 & 0 \\ 0 & 7 & 5 & 2 \end{bmatrix} = \begin{bmatrix} 2 & \frac{1}{2} & -\frac{3}{2} & 0 \\ 0 & -\frac{7}{2} & -\frac{5}{2} & -1 \end{bmatrix}$$

$$10A + 2B = \begin{bmatrix} 20 & 70 & 0 & -10 \\ 50 & -30 & 20 & 20 \end{bmatrix} + \begin{bmatrix} -8 & -2 & 6 & 0 \\ 0 & 14 & 10 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 12 & 68 & 6 & -10 \\ 50 & -16 & 30 & 24 \end{bmatrix}$$

Matrix multiplication is defined only if the sizes are compatible:

Examples

	Result
$(3\times 2)\cdot(2\times 5)$	3×5
$(1\times 3)\cdot(3\times 2)$	1×2
$(2\times 2)\cdot(2\times 5)$	2×5

$$\underset{1\times 3}{\begin{bmatrix}1 & 3 & 2\end{bmatrix}}\underset{3\times 1}{\begin{bmatrix}2\\5\\-1\end{bmatrix}} = \underset{1\times 1}{\begin{bmatrix}15\end{bmatrix}} \leftarrow (1)(2)+(3)(5)+(2)(-1)$$

$$\underset{3\times 1}{\begin{bmatrix}2\\5\\-1\end{bmatrix}}\underset{1\times 3}{\begin{bmatrix}1 & 3 & 2\end{bmatrix}} = \underset{3\times 3}{\begin{bmatrix}2 & 3 & 4\\5 & 15 & 10\\-1 & -3 & -2\end{bmatrix}}$$

ex: $A = \begin{bmatrix}1 & 2\\3 & 4\end{bmatrix}$ $B = \begin{bmatrix}0 & 0\\1 & 1\end{bmatrix}$

$$AB = \begin{bmatrix}1 & 2\\3 & 4\end{bmatrix}\begin{bmatrix}0 & 0\\1 & 1\end{bmatrix} = \begin{bmatrix}2 & 2\\4 & 4\end{bmatrix}$$

$$BA = \begin{bmatrix}0 & 0\\1 & 1\end{bmatrix}\begin{bmatrix}1 & 2\\3 & 4\end{bmatrix} = \begin{bmatrix}0 & 0\\4 & 6\end{bmatrix}$$

$AB \neq BA$

<u>Fact</u>: the $n \times n$ "identity matrix"

has the form $\begin{bmatrix} 1 & 0 & 0 & \cdots & 0 \\ 0 & 1 & 0 & & 0 \\ 0 & 0 & 1 & & 0 \\ & & & \ddots & \\ 0 & 0 & 0 & \cdots & 1 \end{bmatrix} = I$

$$\overset{2\times 2}{\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}} \overset{2\times 3}{\begin{bmatrix} 1 & 2 & 3 \\ 7 & 6 & 5 \end{bmatrix}} = \overset{2\times 3}{\begin{bmatrix} 1 & 2 & 3 \\ 7 & 6 & 5 \end{bmatrix}}$$

$$\overset{2\times 3}{\begin{bmatrix} 1 & 2 & 3 \\ 7 & 6 & 5 \end{bmatrix}} \overset{3\times 3}{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}} = \overset{2\times 3}{\begin{bmatrix} 1 & 2 & 3 \\ 7 & 6 & 5 \end{bmatrix}}$$

<u>ex</u>:
$$\begin{cases} 9x + 4y = 7 \\ 2x + y = 3 \end{cases}$$

$$\begin{bmatrix} 9 & 4 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 7 \\ 3 \end{bmatrix}$$

<u>form</u>:
$$A \underline{X} = B$$
$$A^{-1} A \underline{X} = A^{-1} B$$
$$I \underline{X} = A^{-1} B$$
$$\underline{X} = A^{-1} B$$

Remark: Not all matrices A have an inverse, A^{-1},

But this A does.

$$A = \begin{bmatrix} 9 & 4 \\ 2 & 1 \end{bmatrix} \qquad A^{-1} = \begin{bmatrix} 1 & -4 \\ -2 & 9 \end{bmatrix}$$

check: $A^{-1}A = \begin{bmatrix} 1 & -4 \\ -2 & 9 \end{bmatrix} \begin{bmatrix} 9 & 4 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

so $\begin{bmatrix} x \\ y \end{bmatrix} = \bar{X} = A^{-1}B$

$$= \begin{bmatrix} 1 & -4 \\ -2 & 9 \end{bmatrix} \begin{bmatrix} 7 \\ 3 \end{bmatrix} = \begin{bmatrix} -5 \\ 13 \end{bmatrix}$$

$$\boxed{(x, y) = (-5, 13)}$$

check: $\begin{cases} 9x + 4y = 7 \\ 2x + y = 3 \end{cases}$

$9(-5) + 4(13) = 7$ ✓

$2(-5) + 13 = 3$ ✓

Remark [after end of class]: For 2×2 matrices $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$,

$A^{-1} = \dfrac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$ provided that $ad - bc \neq 0$. If $ad - bc = 0$, then A^{-1} does not exist.