8.4 Systems of Three equations and three variables

8) Is $(x, y, z) = (-1, -3, 2)$ a solution of

$$\begin{cases} x - y + z = 4 & (1) \\ x - 2y - z = 3 & (2) \\ 3x + 2y - z = 1 & (3) \end{cases}$$

Does the ordered triple $(-1, -3, 2)$ satisfy (1)?

$(-1) - (-3) + 2 = -1 + 3 + 2 \overset{\checkmark}{=} 4$ yes.

Does the triple satisfy (2)?

$(-1) - 2(-3) - (2) = -1 + 6 - 2 \overset{\checkmark}{=} 3$ yes.

Does the triple satisfy (3)?

$3(-1) + 2(-3) - (2) = -3 - 6 - 2 = -11 \neq 1$ No.

Overall answer: $\boxed{\text{No}}$

ex: [Solving by elimination]

8.4 10)

$$\begin{cases} x + y - z = 0 & (1) \\ 2x - y + z = 3 & (2) \\ -x + 5y - 3z = 2 & (3) \end{cases}$$

Idea: Go from 3 eqns/3 variables to 2 eqns/2 variables to 1 eqn/1 variable.

Let's eliminate x, two ways.

$$\begin{array}{ll} (1): & x + y - z = 0 \\ (3): & -x + 5y - 3z = 2 \\ \hline (4) & 6y - 4z = 2 \end{array}$$

$$\begin{array}{ll} 2\cdot(3): & -2x + 10y - 6z = 4 \\ (2): & 2x - y + z = 3 \\ \hline (5) & 9y - 5z = 7 \end{array}$$

$$\begin{array}{l} (4) \\ (5) \end{array} \begin{cases} 6y - 4z = 2 \\ 9y - 5z = 7 \end{cases} \rightarrow \begin{array}{ll} -\frac{3}{2}(4): & -9y + 6z = -3 \\ (5): & 9y - 5z = 7 \\ \hline (6) & \boxed{z = 4} \end{array}$$

10 cont'd) Now we "back-substitute" to find y and x.

Use (5): $9y - 5(4) = 7$

$9y - 20 = 7$

$9y = 27$

$\boxed{y = 3}$

Use (1): $x + (3) - (4) = 0$

$x - 1 = 0$

$\boxed{x = 1}$

$$\boxed{(x, y, z) = (1, 3, 4)}$$

22)
$$\begin{cases} x - y + z = 4 & (1) \\ 5x + 2y - 3z = 2 & (2) \\ 4x + 3y - 4z = -2 & (3) \end{cases}$$

$2 \cdot (1)$: $2x - 2y + 2z = 8$

(2): $5x + 2y - 3z = 2$

(4): $7x - z = 10$

$3 \cdot (1)$: $3x - 3y + 3z = 12$

(3): $4x + 3y - 4z = -2$

Whoa! → (5): $7x - z = 10$

$$\begin{cases} 7x - z = 10 & (4) \\ 7x - z = 10 & (5) \end{cases}$$

$-1 \cdot (4)$: $-7x + z = -10$

(5): $7x - z = 10$

$0 = 0$

Dependent system: Infinitely many solutions

18)
$$\begin{cases} 4x + y + z = 17 & (1) \\ x - 3y + 2z = -8 & (2) \\ 5x - 2y + 3z = 5 & (3) \end{cases}$$

$-4 \cdot (2)$: $-4x + 12y - 8z = 32$

(1): $4x + y + z = 17$

(4) $13y - 7z = 49$

$-5 \cdot (2)$: $-5x + 15y - 10z = 40$

(3): $5x - 2y + 3z = 5$

(5) $13y - 7z = 45$

$$\begin{cases} 13y - 7z = 49 & (4) \\ 13y - 7z = 45 & (5) \end{cases}$$

$-1 \cdot (5)$: $-13y + 7z = -45$

(4): $13y - 7z = 49$

$0 = 4$

There is NO SOLUTION

The system is inconsistent.

13) ① $3x + 4y - 3z = 4$

② $5x - y + 2z = 3$

③ $x + 2y - z = -2$

$4 \cdot$ ②: $20x - 4y + 8z = 12$

①: $3x + 4y - 3z = 4$

④ $23x + 5z = 16$

$2 \cdot$ ② $10x - 2y + 4z = 6$

③ $x + 2y - z = -2$

⑤: $11x + 3z = 4$

④ $23x + 5z = 16$

⑤ $11x + 3z = 4$

ANSWER: $(x, y, z) = \boxed{(2, -5, -6)}$

$3 \cdot$ ④: $69x + 15z = 48$

$-5 \cdot$ ⑤ $-55x - 15z = -20$

⑥ $14x = 28 \Rightarrow \boxed{x = 2}$

Use ⑤: $11(2) + 3z = 4$

$22 + 3z = 4$

$3z = -18$

$\boxed{z = -6}$

Use ②: $5(2) - y + 2(-6) = 3$

$10 - y - 12 = 3$

$-y - 2 = 3$

$-y = 5$

$\boxed{y = -5}$