8.6 Elimination using matrices

ex: $2x + 3y = 7$ is represented by the "augmented matrix"

$3x + 4y = 1$

$$\begin{bmatrix} 2 & 3 & 7 \\ 3 & 4 & 1 \end{bmatrix}$$

ex: Here as an augmented matrix of a system of equation. what is the system of equations?

$$\begin{bmatrix} 2 & 5 & -3 & 4 \\ 1 & -1 & 0 & 7 \\ 0 & 2 & 3 & 0 \end{bmatrix}$$

$$\begin{cases} 2x + 5y - 3z = 4 \\ x - y = 7 \\ 2y + 3z = 0 \end{cases}$$

Elementary Row Operations

Each of these row operations produces a "row-~~equivalant~~ equivalent matrix."

["Row equivalent" means that the systems represented by the matrices are equivalent, i.e. has the same solution set.]

1. Swap any two rows, e.g. $R_1 \leftrightarrow R_3$
2. Multiply a row by a (nonzero) constant, e.g. $-3R_2$.
3. Add a multiple of (a copy of) one row to another e.g. $5R_1 + R_2 \rightarrow R_2$.

ex:

$$\begin{aligned} 2x + y + 3z &= 1 \\ x + 2y + 4z &= 6 \\ -2x \quad\quad - z &= 7 \end{aligned}$$

has as its augmented matrix:

$$\begin{bmatrix} 2 & 1 & 3 & 1 \\ 1 & 2 & 4 & 6 \\ -2 & 0 & -1 & 7 \end{bmatrix}$$

Goal: A matrix in "row echelon form"

$$\begin{bmatrix} 1 & * & * & * \\ 0 & 1 & * & * \\ 0 & 0 & 1 & * \end{bmatrix}$$

$R_1 \leftrightarrow R_2$

$$\begin{bmatrix} 1 & 2 & 4 & 6 \\ 2 & 1 & 3 & 1 \\ -2 & 0 & -1 & 7 \end{bmatrix} \sim$$

$$-2R_1 + R_2 \to R_2 \quad \begin{bmatrix} 1 & 2 & 4 & 6 \\ 0 & -3 & -5 & -11 \\ -2 & 0 & -1 & 7 \end{bmatrix}$$

$$2R_1 + R_3 \to R_3 \quad \begin{bmatrix} 1 & 2 & 4 & 6 \\ 0 & -3 & -5 & -11 \\ 0 & 4 & 7 & 19 \end{bmatrix}$$

$$R_2 + R_3 \to R_3 \quad \begin{bmatrix} 1 & 2 & 4 & 6 \\ 0 & -3 & -5 & -11 \\ 0 & 1 & 2 & 8 \end{bmatrix}$$

$$R_2 \leftrightarrow R_3 \quad \begin{bmatrix} 1 & 2 & 4 & 6 \\ 0 & 1 & 2 & 8 \\ 0 & -3 & -5 & -11 \end{bmatrix}$$

$$3R_2 + R_3 \to R_3 . \quad \begin{bmatrix} 1 & 2 & 4 & 6 \\ 0 & 1 & 2 & 8 \\ 0 & 0 & 1 & 13 \end{bmatrix}$$

← Row-echelon form

$$\begin{cases} x + 2y + 4z = 6 \\ \quad\;\; y + 2z = 8 \\ \qquad\quad\; z = 13 \end{cases}$$

← The corresponding system of equations.

Now do back-substitution:

$$y + 2(13) = 8$$
$$y + 26 = 8$$
$$y = 8 - 26$$
$$y = -18$$

$$x + 2(-18) + 4(13) = 6$$
$$x - 36 + 52 = 6$$
$$x + 16 = 6$$
$$x = -10$$

$$(x, y, z) = (-10, -18, 13)$$

<u>check</u>:

$$2(-10) + (-18) + 3(13) = -20 - 18 + 39 = 1 \quad \checkmark$$
$$-10 + 2(-18) + 4(13) = -10 - 36 + 52 = 6 \quad \checkmark$$
$$-2(-10) \qquad - (13) = 20 - 13 = 7 \quad \checkmark$$

<u>ex</u>: [How to detect in<u>consist</u>ent systems]

$$\begin{aligned} x + 3y &= 7 \\ -2x - 6y &= 15 \end{aligned} \qquad \begin{bmatrix} 1 & 3 & 7 \\ -2 & -6 & 15 \end{bmatrix}$$

$$2R_1 + R_2 \to R_2 \begin{bmatrix} 1 & 3 & 7 \\ 0 & 0 & 29 \end{bmatrix} \qquad \begin{cases} x + 3y = 7 \\ 0 = 29 \end{cases}$$

Inconsistent system.

<u>ex</u>: $\begin{cases} x + 3y = 7 \\ -2x - 6y = -14 \end{cases} \quad \begin{bmatrix} 1 & 3 & 7 \\ -2 & -6 & -14 \end{bmatrix} \quad 2R_1 + R_2 \to R_2 \begin{bmatrix} 1 & 3 & 7 \\ 0 & 0 & 0 \end{bmatrix}$

Dependent system

<u>Warmup/Hint</u> for the quiz

A warehouse contains parts to manufacture unicycles, bicycles, and tricycles. The warehouse holds 9 seats, 19 wheels, 6 sets of handlebars.

How many unicycles, bicycles, tricycles can we manufacture from these parts?

<u>NOTE</u>: A <u>unicycle</u> has 1 seat, 1 wheel, No handlebars
A <u>bicycle</u> has 1 seat, 2 wheels, 1 set of handlebars
A <u>tricycle</u> has 1 seat, 3 wheels, 1 set of handlebars.

Let x = number of unicycles
y = number of bicycles
z = number of tricycles

$$\begin{array}{lll} (1) & \text{Seats:} & x + y + z = 9 \\ (2) & \text{wheels:} & x + 2y + 3z = 19 \\ (3) & \text{Sets of bars:} & y + z = 6 \end{array}$$

$$\begin{array}{lrcrcrcr} -1\cdot(1) & -x & - & y & - & z & = & -9 \\ (2) & x & + & 2y & + & 3z & = & 19 \\ \hline (4) & & & y & + & 2z & = & 10 \\ -1\cdot(3) & & & -y & - & z & = & -6 \\ \hline (5) & & & & & z & = & 4 \end{array}$$

$z = 4 \Rightarrow \boxed{4 \text{ tricycles}}$

Use (3): $y + 4 = 6 \Rightarrow y = 2 \Rightarrow \boxed{2 \text{ bikes}}$

Use (1): $x + 2 + 4 = 9 \Rightarrow x = 3 \Rightarrow \boxed{3 \text{ unicycles}}$

Intro to 8.7

ex [2x2 "determinant"]

$$\begin{vmatrix} 3 & 5 \\ 7 & 12 \end{vmatrix} = (3)(12) - (7)(5)$$

$$= 36 - 35 = 1$$

ex:

$$\begin{vmatrix} 7 & -2 \\ 4 & 8 \end{vmatrix} = (7)(8) - (4)(-2)$$

$$= 56 + 8 = 64$$

ex:

$$\begin{vmatrix} 2 & 7 \\ 4 & 14 \end{vmatrix} = (2)(14) - (4)(7)$$

$$= 28 - 28 = 0$$

ex:

$$\begin{vmatrix} -3 & -5 \\ -2 & -1/2 \end{vmatrix} = (-3)\left(-\frac{1}{2}\right) - (-2)(-5)$$

$$= \frac{3}{2} - 10 = \frac{3}{2} - \frac{20}{2} = -\frac{17}{2}$$

Defn:

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$$