

9.3 Absolute Value Equations and Inequalities

ex: $|7| = 7$ $|-3| = 3$
 $|0| = 0$

Definition: $|x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$

$|-3| = -(-3) = 3$ because $-3 < 0$

$|7| = 7$ because $7 \geq 0$

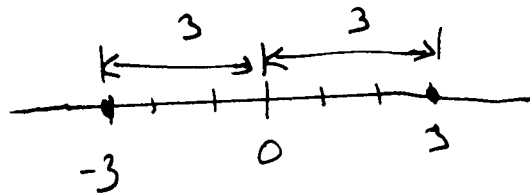
ex: Solve $|x| = 3$ has two solutions

$x = 3$ or $x = -3$

ex: Solve $|x| = 0$ has one solution

$x = 0$

ex: Solve $|x| = -2$ has no solution



ex: $|x| + 2 = 5$
 $\quad \quad -2 \quad -2$

$$|x| = 3$$

$$x = 3 \text{ or } -3$$

ex: $|x+2| = 5$

$$|\underline{x}| = 5 \quad \text{where } \underline{x} = x+2$$

$$\underline{x} = 5 \quad \text{or} \quad \underline{x} = -5$$

$$x+2 = 5 \quad \text{or} \quad x+2 = -5$$

$$x = 3 \quad \text{or} \quad x = -7$$

ex: $|x-1| = 10$

$$x-1 = 10 \quad \text{OR} \quad x-1 = -10$$

$$x = 11 \quad \text{OR} \quad x = -9$$

check:

$$|11-1| = |10|$$

$$= 10$$

$$|-9-1| = |-10|$$

$$= 10$$

$$\underline{\text{ex:}} \quad |x-2| + 2 = 8$$

-2 -2

$$|x-2| = 6$$

↙ or ↘

$$x-2 = 6$$

+2 +2

$$x = 8$$

$$x-2 = -6$$

+2 +2

$$x = -4$$

$$\underline{\text{ex:}} \quad |x-2| + 8 = 2$$

-8 -8

$$|x-2| = -6$$

↑
nonnegative
number

↑
negative
number

No solution.

$$32) \quad |2y| - 5 = 13$$

+5 +5

$$|2y| = 18$$

↙

$$2y = 18$$

$$y = 9$$

or

↘

$$2y = -18$$

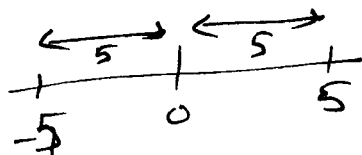
$$y = -9$$

or

Remark : $|3| = |3|$ because $3=3$

$|-7| = |-7|$ because $-7=-7$

$|-5| = |5|$ even though $-5 \neq 5$



ex : $|x-7| = |2x+1|$
47)

↙ or ↘

$$\begin{array}{r} x-7 = 2x+1 \\ +7 \quad +7 \\ \hline \end{array}$$

$$\begin{array}{r} x = 2x+8 \\ -2x \quad -2x \\ \hline \end{array}$$

$$-x = 8$$

$$x = -8$$

①

$$-(x-7) = 2x+1$$

$$-x+7 = 2x+1$$

$$+x \quad +x$$

$$7 = 3x+1$$

$$-1 \quad -1$$

$$(2) \quad 6 = 3x$$

$$2 = x$$

check ① : $|(-8)-7| \stackrel{?}{=} |2(-8)+1|$

$$|-15| = |-15| \checkmark$$

check ② : $|2(2)+1| \stackrel{?}{=} |2-7|$

$$|5| = |-5| \checkmark$$

$$5 = 5 \checkmark$$

(5)

TRY THESE

$$(1) \quad |2x - 3| = 4$$

solution for (1):

$$2x - 3 = 4 \quad \text{or} \quad 2x - 3 = -4$$

$$+3 \quad +3$$

$$2x = 7$$

$$\boxed{x = 7/2}$$

$$+3 \quad +3$$

$$2x = -1$$

$$\boxed{x = -1/2}$$

$$(2) \quad |x| + 3 = 3$$

$$\text{solution to (2): } |x| = 0$$

$$\boxed{x = 0}$$

$$(3) \quad 5|x| + 2 = 17$$

$$\begin{array}{r} -2 \quad -2 \\ 5|x| = 15 \end{array}$$

solution to (3):

$$5|x| = 15$$

$$|x| = 3$$

$$\boxed{x = 3} \text{ or } \boxed{x = -3}$$

$$(4) \quad |3x + 2| = |x - 6|$$

solution to (4):

$$3x + 2 = x - 6$$

$$-x \quad -x$$

$$2x + 2 = -6$$

$$-2 \quad -2$$

$$2x = -8$$

$$\boxed{x = -4}$$

$$3x + 2 = -(x - 6)$$

$$3x + 2 = -x + 6$$

$$+x \quad +x$$

$$4x + 2 = 6$$

$$-2 \quad -2$$

$$4x = 4$$

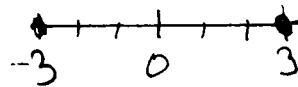
$$\boxed{x = 1}$$

or

(1) GRAPH $|x| = 3$

means

$$x = 3 \text{ or } x = -3$$



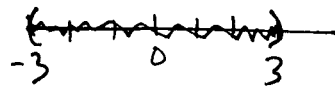
$$\{-3, 3\}$$

(2) Graph $|x| < 3$

means

$$-3 < x < 3$$

so $-3 < x$ AND $x < 3$

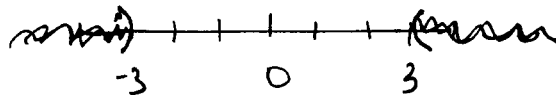


$$(-3, 3)$$

(3) Graph $|x| > 3$

$$(-\infty, -3) \cup (3, \infty)$$

$$x > 3 \text{ OR } x < -3$$

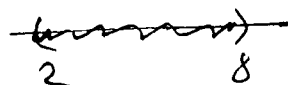


ex: $|x - 5| < 3$ is equivalent to:

$$\begin{array}{ccc} -3 < x - 5 < 3 \\ +5 & +5 & +5 \end{array}$$

$$2 < x < 8$$

so soln set = $(2, 8)$



ex: $|x - 5| > 3$

$$\begin{array}{ccc} x - 5 > 3 & \text{OR} & x - 5 < -3 \\ +5 & +5 & +5 & +5 \end{array}$$

$$x > 8 \text{ OR } x < 2$$

$$(-\infty, 2) \cup (8, \infty)$$



INCORRECT:

$$2 > x > 8 \text{ This would mean } 2 > x \text{ AND } x > 8$$