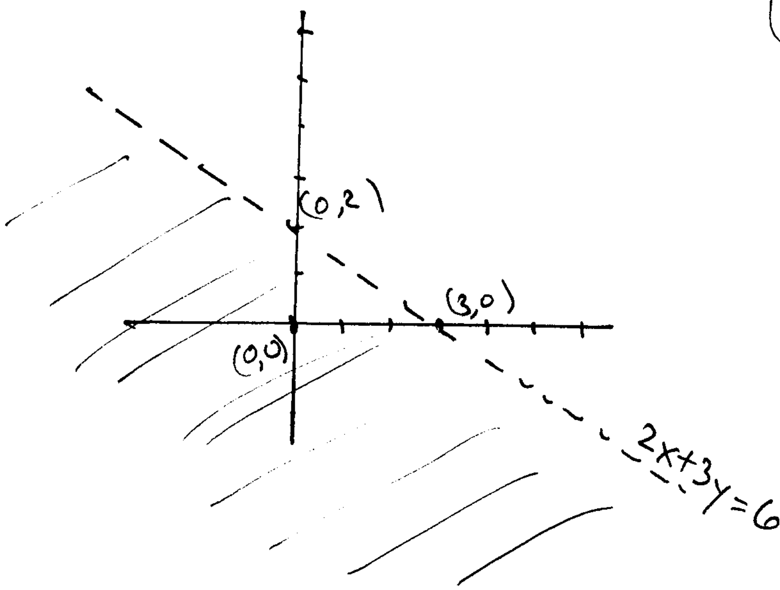
Math 96 Tue 22 SEP 2015

9.4 Inequalities in two variables

ex: $2x+3y<6$



Step 1 Graph $2x+3y=6$

x	y
3	0
0	2

Step 2 Test point

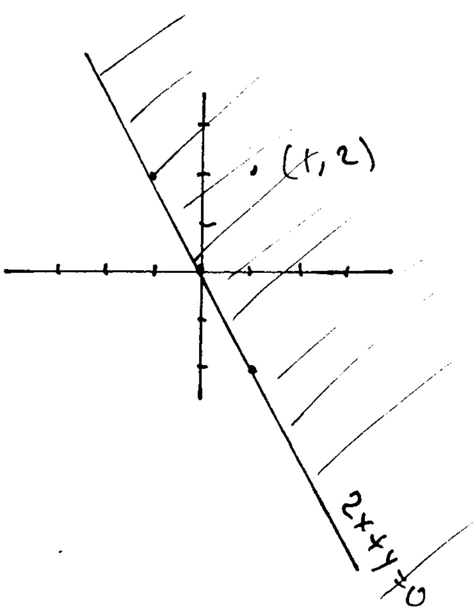
$(x,y)=(0,0)$

Does $(0,0)$ satisfy the inequality?

$2(0)+3(0)\overset{?}{<}6$

Yes!

ex: $2x+y\geq 0$



Step 1 Graph $2x+y=0$

$y=-2x=-\frac{2}{1}x+0$

Step 2 Test pt: $(x,y)=(1,2)$

$2(1)+(2)=4\geq 0$ ✓

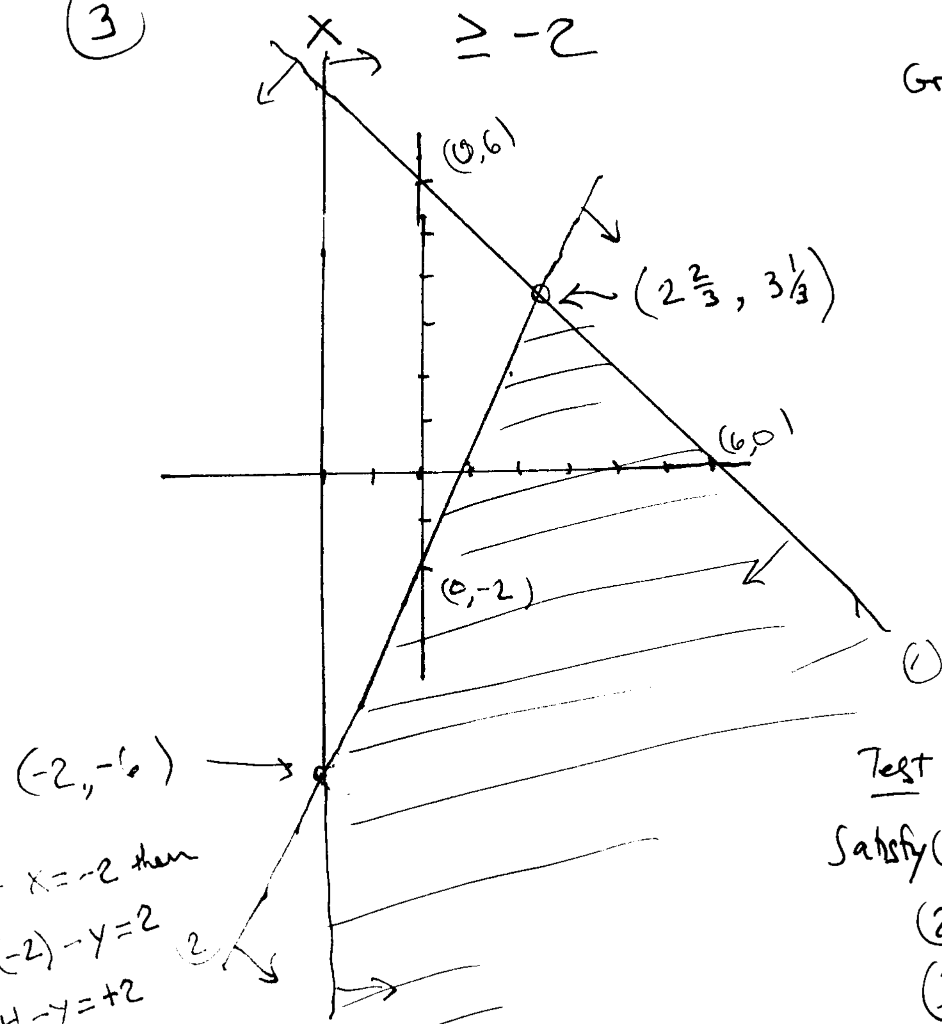
Yes.

ex [System of inequalities] Graph this system, and find coordinates of any vertices formed.

(1) $x + y \le 6$

(2) $2x - y \ge 2$

(3) $x \ge -2$



If $x=-2$ then
$2(-2)-y=2$
$-4-y=+2$
$-y=6$
$y=-6$

Graph (1) $x+y=6$

x	y
6	0
0	6

(2) $2x-y=2$ or
$2x=y+2$ or
$2x-2=y$

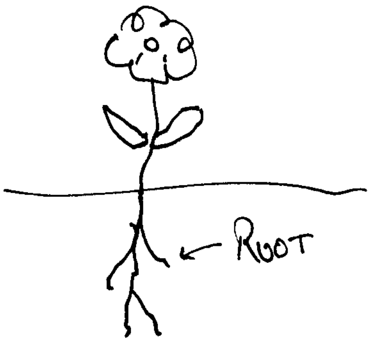
(3) $x=-2$

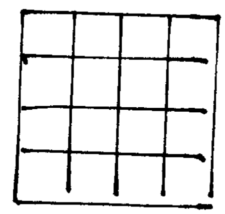
Test using $(x,y)=(0,0)$
Satisfy (1)? Yes.
(2)? No.
(3)? Yes.

10.1 Radical Expressions and Functions

ex: $\sqrt{49} = 7$ because $7^2 = 49$

ex: $\sqrt{16} = 4$ why is this called a "square root?"



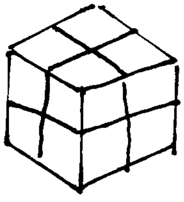


16 small squares arranged into a large square.
What is the length of one side of the square?

ex: $\sqrt[3]{125} = 5$ because $5^3 = 125$

$\sqrt[3]{8} = 2$

$\sqrt[6]{1{,}000{,}000} = 10$



$\sqrt[5]{32} = 2$ because $2^5 = 32$

$\sqrt[3]{-8} = -2$ because $(-2)^3 = -8$

$\sqrt{-49}$ is not a real number (its $7i$)

Remark: $(-4)^2 = 16$ so 4 and -4 are both square roots of 16,
but $\sqrt{16}$ means 4 not -4; $-\sqrt{16} = -4$.

ex $\sqrt[4]{81} = 3$ because $3^4 = 81$

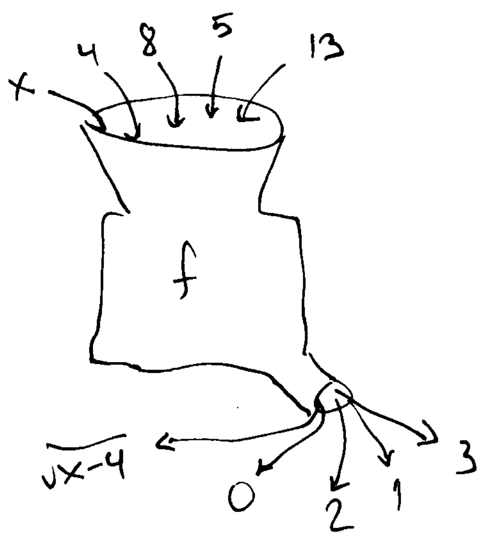
Note: -3 is also a 4th root of 81, but $\sqrt[4]{81}$ means 3.

ex: $\sqrt[4]{-81}$ is not real.

ex: $\sqrt[5]{-32} = -2$ because $(-2)^5 = -32$.

ex: [Function] $f(x) = \sqrt{x-4}$

Remark: Informally a function is a machine that eats numbers and then spews out numbers in response.



$f(13) = \sqrt{13-4} = \sqrt{9} = 3$

$f(5) = \sqrt{5-4} = \sqrt{1} = 1$

$f(8) = \sqrt{8-4} = \sqrt{4} = 2$

$f(4) = \sqrt{4-4} = \sqrt{0} = 0$

We cannot plug in 0. $f(0) = \sqrt{0-4} = \sqrt{-4}$ is not real

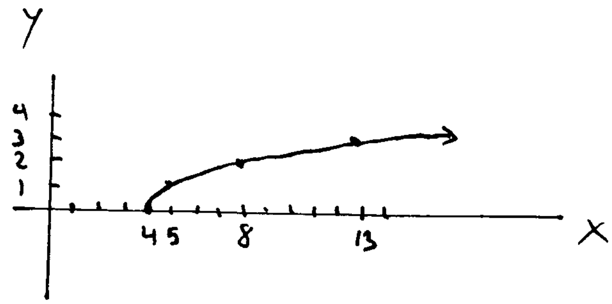
What numbers can we plug.

Answer: Solve $x - 4 \geq 0$, so $x \geq 4$.

The domain of this function (the set of numbers you're allowed to input into this function) $= \{ x \mid x \geq 4 \} = [4, \infty)$

Graph of f

x	$y=\sqrt{x-4}$
4	0
5	1
8	2
13	3



domain of f = "shadow" of the graph onto the x-axis

$= [4, \infty)$

range of f [set of outputs of f] = "shadow" onto the y-axis

$= [0, \infty)$

10.2 Rational numbers as exponents

Q: What numbers can appear as exponents? A: (so far) Any integer.

$2^3 = 2\cdot2\cdot2 = 8$

$2^2 = 2\cdot2 = 4$

$2^1 = 2 = 2$

$2^0 = 1$

$2^{-1} = 1/2$

$2^{-2} = 1/4$

$2^{-3} = 1/8$

$$\frac{2^5}{2^2} = \frac{\cancel{2}\cdot\cancel{2}\cdot2\cdot2\cdot2}{\cancel{2}\cdot\cancel{2}} = 2^{5-2} = 2^3 = 8$$

$\cancel{= 8}$

what should 2^{-3} mean?

$$\frac{2^2}{2^5} = \frac{2\cdot2}{2\cdot2\cdot2\cdot2\cdot2} = \frac{1}{2^3} = \frac{1}{8}$$

BUT if we want properties of exponents to work the usual way, we want

$$\frac{2^2}{2^5} = 2^{2-5} = 2^{-3}$$

So this another reason we want $2^{-3} = \frac{1}{8}$.

Q: What *should* $9^{1/2}$ mean?

Note: $(2^3)^2 = (2\cdot2\cdot2)^2 = (2\cdot2\cdot2)\cdot(2\cdot2\cdot2)$
$= 2^{2\cdot3} = 2^6 = 64$

$$(2^{-2})^4 = \left(\frac{1}{2\cdot2}\right)^4 = \frac{1}{2\cdot2}\cdot\frac{1}{2\cdot2}\cdot\frac{1}{2\cdot2}\cdot\frac{1}{2\cdot2}$$
$$= \frac{1}{2^8} = 2^{-8} = 2^{(-2)\cdot(4)}$$

Let $x = 9^{1/2}$. Then we *ought* have that

$$x^2 = (9^{1/2})^2 = 9^{(\frac{1}{2})\cdot(2)} = 9^1 = 9$$

So x *should* have the property that $x^2 = 9$.

So x could be 3 or -3.

DEFINE: $9^{1/2} = 3 = \sqrt{9}$

$25^{1/2} = 5$, $49^{1/2} = 7$, $100^{1/2} = 10$

$\left(\frac{1}{4}\right)^{1/2} = \sqrt{\frac{1}{4}} = \frac{1}{2}$.

Defn: $\boxed{a^{1/2} = \sqrt{a}}$ So $(-16)^{1/2}$ is not a real number.

but $81^{1/2} = 9$.

Q: What <u>should</u> $8^{1/3}$ mean?

Let $x = 8^{1/3}$, then we ought to have

$$x^3 = (8^{1/3})^3 = 8^{\frac{1}{3}\cdot 3} = 8^1 = 8$$

so $x = 8^{1/3} = \sqrt[3]{8} = 2$.

<u>Defn:</u> $\boxed{a^{1/3} = \sqrt[3]{a}}$

<u>ex:</u> $1000^{1/3} = 10$

$125^{1/3} = 5$

$27^{1/3} = 3$

$(-8)^{1/3} = \sqrt[3]{-8} = -2$

<u>Defn</u>: $\boxed{a^{1/n} = \sqrt[n]{a}}$

<u>ex:</u> $1{,}000{,}000^{1/6} = 10$ because

$$1000000^{1/6} = (10^6)^{1/6} = 10^{6\cdot\left(\frac{1}{6}\right)} = 10^1 = 10$$

<u>ex</u>: $64^{1/3} = (4\cdot 4\cdot 4)^{1/3} = (4^3)^{1/3} = 4$

<u>ex</u>: ~~$49^{-1/3}$~~

Q: What should $8^{2/3}$ mean?

$8^{2/3}$ <u>ought</u> to mean $8^{\frac{1}{3}\cdot 2} = (8^{1/3})^2 = 2^2 = 4$.

$$25^{3/2} = (25^{1/2})^3 = 5^3 = 125$$

Defn: $$\boxed{a^{\frac{m}{n}} = \left(\sqrt[n]{a}\right)^m}$$

ex:

$$8^{\frac{5}{3}} = \left(8^{1/3}\right)^5 = 2^5 = 32$$

$$27^{2/3} = \left(27^{1/3}\right)^2 = 3^2 = 9$$

$$16^{-1/2} = \left(16^{1/2}\right)^{-1} = 4^{-1} = \frac{1}{4}$$

$$32^{-3/5} = ?$$

Anatomy of a fractional exponent

reciprocal

3^{rd} power

$$32^{-\frac{3}{5}}$$

5^{th} root

$$= \left[\left(32^{1/5}\right)^3\right]^{-1} = \left[2^3\right]^{-1} = 8^{-1} = \frac{1}{8}$$

Remark: Any other order would also work.

$$\left[\left(32^3\right)^{1/5}\right]^{-1} = \left[32{,}768^{1/5}\right]^{-1} = 8^{-1} = \frac{1}{8}$$

ex:

$$(-8)^{-5/3} = \left[\left(\sqrt[3]{-8}\right)^5\right]^{-1} = \left[(-2)^5\right]^{-1} = [-32]^{-1} = \frac{-1}{32}$$

ex:
(If you handle this you can handle anything.)

$$\left(-\frac{8}{27}\right)^{-2/3} = \left[\left(-\frac{8}{27}\right)^{-1}\right]^{2/3} = \left(\left[-\frac{27}{8}\right]^{1/3}\right)^2 = \left(-\frac{3}{2}\right)^2 = \frac{9}{4}$$