10.2 Fractional Exponents (cont'd)

ex: $(x^2)^3 = x^6$

$(x^{1/2})^{2/3} = x^{1/3}$

ex: $x^3 \cdot x^5 = x^8$

$x^{2/3} \cdot x^{4/3} = x^{6/3} = x^2$ because $\frac{2}{3} + \frac{4}{3} = \frac{6}{3} = 2$

Less obvious:

$\sqrt[3]{x^2} \cdot \sqrt[3]{x^4} = x^2$

ex: $\frac{x^7}{x^3} = x^4$

$\frac{x^{3/2}}{x^{-1/2}} = x^{\frac{3}{2} - (-\frac{1}{2})} = x^{\frac{3}{2} + \frac{1}{2}} = x^{4/2} = x^2$

OR: $\frac{x^{3/2}}{x^{-1/2}} = \frac{x^{3/2} \cdot x^{1/2}}{1} = x^{3/2} \cdot x^{1/2} = x^{3/2 + 1/2} = x^2$

ex: $(x^2 y^4)^3 = (x^2)^3 (y^4)^3 = x^6 y^{12}$

$(x^{1/2} y^{2/3})^{\frac{6}{7}} = (x^{1/2})^{\frac{6}{7}} \cdot (y^{2/3})^{6/7} = x^{3/7} y^{\frac{4}{7}}$

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(2)

Try these : Simplify

ANSWERS:

70) $5^{\frac{1}{4}} \cdot 5^{\frac{1}{8}}$ $\qquad 5^{\frac{3}{8}}$

72) $\dfrac{8^{7/11}}{8^{-2/11}} = 8^{\frac{7}{11}} \cdot 8^{2/11} = 8^{9/11}$

78) $x^{3/4} \cdot x^{1/3} = x^{\frac{3}{4}+\frac{1}{3}} = x^{\frac{9}{12}+\frac{4}{12}} = x^{13/12}$

82) $\left(x^{-1/3}\, y^{2/5}\right)^{1/4} = \left(x^{-1/3}\right)^{1/4} \cdot \left(y^{2/5}\right)^{1/4}$

$= \dfrac{x^{-1/12} \cdot y^{\frac{1}{10}}}{1} = \dfrac{y^{1/10}}{x^{1/12}}$

Use rational exponents to simplify these radical expressions

83) $\sqrt[9]{x^3} = x^{3/9} = x^{1/3} = \sqrt[3]{x}$

90) $\left(\sqrt[3]{ab}\right)^{15} = (ab)^{\frac{15}{3}} = (ab)^5 = a^5 b^5$

TRY these:

92) $\sqrt[8]{(3x)^2} = (3x)^{\frac{2}{8}} = (3x)^{\frac{1}{4}} = \sqrt[4]{3x}$

96) $\sqrt[6]{\sqrt{n}} = \left(n^{1/2}\right)^{\frac{1}{6}} = n^{1/12} = \sqrt[12]{n}$

98) $\sqrt{(ab)^6} = (ab)^{\frac{6}{2}} = (ab)^3 = a^3 b^3$

<u>warning</u>: Do not not apply exponent rules which don't exist.

$$(x^2+y^2)^2 \quad \underline{\text{is}}\ \underline{\text{NOT}} \quad (x^2)^2+(y^2)^2$$

It <u>is</u>
$$\begin{aligned}(x^2+y^2)^2 &= (x^2+y^2)(x^2+y^2)\\ &= (x^2)^2+2x^2y^2+(y^2)^2\\ &= x^4+2x^2y^2+y^4\end{aligned}$$

<u>Likewise</u> $(x^2+y^2)^{1/2} = \sqrt{x^2+y^2}$ is <u>NOT</u> $x+y$.

For example: $\sqrt{3^2+4^2} = \sqrt{25} = 5 \neq 3+4 = 7$
$= (3^2+4^2)^{1/2}$

10.3 Multiplying Radicals

<u>ex</u>: $\sqrt{4}\,\sqrt{9} = \sqrt{4\cdot 9}$
$2\cdot 3 = 6$

$$\boxed{\sqrt{a}\,\sqrt{b} = \sqrt{ab}}$$

$$\boxed{\sqrt[n]{a}\,\sqrt[n]{b} = \sqrt[n]{ab}}$$

ex: $\sqrt{12} = \sqrt{4 \cdot 3} = \sqrt{4}\sqrt{3} = 2\sqrt{3}$

ex: $\sqrt{75} = \sqrt{25 \cdot 3} = \sqrt{25}\sqrt{3} = 5\sqrt{3}$

ex: $\sqrt{98} = \sqrt{49 \cdot 2} = \sqrt{49}\sqrt{2} = 7\sqrt{2}$

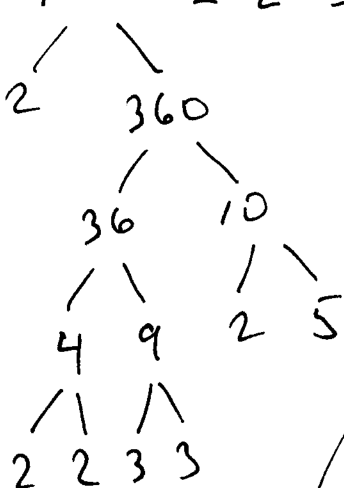
ex: $\sqrt{720} = \sqrt{2^4 \cdot 3^2 \cdot 5} = \sqrt{2^4}\sqrt{3^2}\sqrt{5}$

$= (2^4)^{1/2}(3^2)^{1/2}\sqrt{5}$

$= 2^2 \cdot 3 \cdot \sqrt{5}$

$= 12\sqrt{5}$

$720 = 2^4 \cdot 3^2 \cdot 5$



ex: $\sqrt[3]{32} = \sqrt[3]{2 \cdot 2 \cdot 2 \cdot 2 \cdot 2}$

$= \sqrt[3]{2^3 \cdot 2^2}$

$= \sqrt[3]{2^3} \cdot \sqrt[3]{2^2} = \sqrt[3]{8}\sqrt[3]{4}$

$= 2\sqrt[3]{4}$

TRY THIS ex: Simplify: $\sqrt[3]{250} = \sqrt[3]{2 \cdot 5 \cdot 5 \cdot 5} = \sqrt[3]{5^3 \cdot 2}$

$= \sqrt[3]{5^3} \cdot \sqrt[3]{2}$

$= 5 \cdot \sqrt[3]{2}$

experiment: $\sqrt{3^2} = 3$ $\quad \sqrt{5^2} = 5$ $\quad \sqrt{18^2} = 18$

$\sqrt{(-10)^2} = \sqrt{100} = 10$ $\quad \sqrt{(-2)^2} = 2$ $\quad \sqrt{(-18)^2} = 18$

Aha! $\boxed{\sqrt{a^2} = |a|}$

experiments: $\sqrt[3]{(-2)^3} = \sqrt[3]{-8} = -2$

$\sqrt[3]{5^3} = \sqrt[3]{125} = 5$

$\sqrt[3]{(-5)^3} = \sqrt[3]{-125} = -5$

Aha! $\boxed{\sqrt[3]{a^3} = a}$

Powers of small integers:

$2^1=2$	$3^1=3$	$4^1=4$	$5^1=5$	$6^1=6$	$7^1=7$
$2^2=4$	$3^2=9$	$4^2=16$	$5^2=25$	$6^2=36$	$7^2=49$
$2^3=8$	$3^3=27$	$4^3=64$	$5^3=125$	$6^3=216$	$7^3=343$
$2^4=16$	$3^4=81$	$4^4=256$	$5^4=625$		
$2^5=32$	$3^5=243$	$4^5=1024$			
$2^6=64$					
$2^7=128$					
$2^8=256$					
$2^9=512$					
$2^{10}=1024$					

ex: $\sqrt[3]{625} = \sqrt[3]{5^4}$

$= \sqrt[3]{5^3 \cdot 5}$

$= \sqrt[3]{5^3}\,\sqrt[3]{5}$

$= 5\sqrt[3]{5}$

ex. Simplify $\sqrt{3}\,\sqrt{27} = \sqrt{3 \cdot 27} = \sqrt{81} = 9$

OR $\sqrt{3}\,\sqrt{27} = \sqrt{3} \cdot \sqrt{9 \cdot 3} = \sqrt{3}\,\sqrt{9}\,\sqrt{3}$

$= \sqrt{3} \cdot 3\sqrt{3} = 3 \cdot \sqrt{3}^2$

$= 3 \cdot 3 = 9$

12) $\sqrt[4]{4}\,\sqrt[4]{10} = \sqrt[4]{40}$

14) $\sqrt{5a}\,\sqrt{6b} = \sqrt{5a \cdot 6b} = \sqrt{30ab}$

Simplify

30) $\sqrt{27} = \sqrt{9}\,\sqrt{3} = 3\sqrt{3}$

32) $\sqrt{75y^5} = \sqrt{25 \cdot y^4 \cdot 3 \cdot y}$

$= \sqrt{25}\,\sqrt{y^4}\,\sqrt{3y} = 5y^2\sqrt{3y}$

TRY THIS 36) $\sqrt{175y^8} = \sqrt{5^2 \cdot 7 y^8} = \sqrt{5^2 (y^4)^2}\,\sqrt{7}$

$= \sqrt{5^2}\,\sqrt{(y^4)^2}\,\sqrt{7} = 5y^4\sqrt{7}$

TRY 38) $\sqrt[3]{27ab^3} = \sqrt[3]{3^3 b^3 \cdot a} = \sqrt[3]{(3b)^3}\,\sqrt[3]{a}$

$= 3b\,\sqrt[3]{a}$

TRY 40) $\sqrt[3]{-32a^6} = \sqrt[3]{-8 \cdot 4}\,\sqrt[3]{a^6} = \sqrt[3]{-8}\,\sqrt[3]{a^6}\,\sqrt[3]{4}$

$= -2a^2\,\sqrt[3]{4}$

10.4 Dividing Radicals

$$\boxed{\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}}$$

ex: $\sqrt{\frac{9}{100}} = \frac{\sqrt{9}}{\sqrt{100}} = \frac{3}{10}$

ex: $\sqrt[3]{\frac{27}{125}} = \frac{\sqrt[3]{27}}{\sqrt[3]{125}} = \frac{3}{5}$

ex: $\sqrt[3]{\frac{16x^5}{27y^6}} = \frac{\sqrt[3]{16x^5}}{\sqrt[3]{27y^6}} = \frac{\sqrt[3]{16x^5}}{3y^2} = \frac{\sqrt[3]{8x^3}\ \sqrt[3]{2x^2}}{3y^2}$

$$= \frac{2x\ \sqrt[3]{2x^2}}{3y^2}$$

ex: $\frac{\sqrt{80}}{\sqrt{5}} = \sqrt{\frac{80}{5}} = \sqrt{16} = 4$

Rationalizing denominators

ex: $\frac{1}{\sqrt{3}} = \frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{\sqrt{3}^2} = \frac{\sqrt{3}}{3}$

ex: $\sqrt{\frac{5}{12}} = \frac{\sqrt{5}}{\sqrt{12}} = \frac{\sqrt{5}}{\sqrt{4}\sqrt{3}} = \frac{\sqrt{5}}{2\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{15}}{2 \cdot 3} = \frac{\sqrt{15}}{6}$

TRY ex: $\sqrt{\frac{7}{20}} = \frac{\sqrt{7}}{\sqrt{20}} = \frac{\sqrt{7}}{\sqrt{4}\sqrt{5}} = \frac{\sqrt{7}}{2\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} = \frac{\sqrt{35}}{2 \cdot 5} = \frac{\sqrt{35}}{10}$

"Simplify" means:

(1) No fractions under radical (2) ~~No radsin~~ No radicals in denominator and (3) No square factors under $\sqrt{}$