Warmup

1. Evaluate $\left(-\frac{27}{8}\right)^{-\frac{2}{3}} = \left(-\frac{8}{27}\right)^{\frac{2}{3}} = \left(-\frac{2}{3}\right)^{2} = \frac{4}{9}$

2. $$\frac{\left(x^{1/2} \cdot x^{1/4}\right)^{1/2}}{x^{-1/4}} = \frac{\left(x^{3/4}\right)^{1/2}}{x^{-1/4}} = \frac{x^{3/8}}{x^{-1/4}}$$

$$= \frac{x^{\frac{3}{8}} \cdot x^{1/4}}{1} = x^{\frac{3}{8}+\frac{2}{8}} = x^{5/8}$$

3. Simplify $\sqrt{75} = \sqrt{25 \cdot 3} = \sqrt{25} \cdot \sqrt{3} = 5\sqrt{3}$

4. Simplify
$$\frac{6}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{6\sqrt{3}}{3} = 2\sqrt{3}$$

5. $$\sqrt[3]{\frac{5}{16}} = \frac{\sqrt[3]{5}}{\sqrt[3]{16}} = \frac{\sqrt[3]{5}}{\sqrt[3]{8}\sqrt[3]{2}} = \frac{\sqrt[3]{5}}{2\sqrt[3]{2}} \cdot \frac{\sqrt[3]{4}}{\sqrt[3]{4}}$$

$$= \frac{\sqrt[3]{5} \cdot \sqrt[3]{4}}{2 \cdot \sqrt[3]{8}} = \frac{\sqrt[3]{5} \cdot \sqrt[3]{4}}{2 \cdot 2} = \frac{\sqrt[3]{20}}{4}$$

10.5 Expressions Containing Several Radical Terms
(The arithmetic of radicals.)

ex: $3\sqrt{5} + 7\sqrt{5} - 2\sqrt{5}$

$= (3 + 7 - 2)\sqrt{5} = 8\sqrt{5}$

ex: $7\sqrt{2} + 4\sqrt{2} + 5\sqrt{3} + 11\sqrt{3} = 11\sqrt{2} + 16\sqrt{3}$

Remark: Compare with $7x + 4x + 5y + 11y = 11x + 16y$

ex: $\sqrt{12} + \sqrt{75} + \sqrt{27}$

$= \sqrt{4}\sqrt{3} + \sqrt{25}\sqrt{3} + \sqrt{9}\sqrt{3} = 2\sqrt{3} + 5\sqrt{3} + 3\sqrt{3}$

$= 10\sqrt{3}$

ex: $5\sqrt{8} + 7\sqrt{50} + 2\sqrt{2}$

$= 5\sqrt{4}\sqrt{2} + 7\sqrt{25}\sqrt{2} + 2\sqrt{2}$

$= 5\cdot 2\sqrt{2} + 7\cdot 5\sqrt{2} + 2\sqrt{2} = 10\sqrt{2} + 35\sqrt{2} + 2\sqrt{2}$

$= 47\sqrt{2}$

TRY THIS: 19) $3\sqrt{45} - 8\sqrt{20}$

$= 3\sqrt{9}\sqrt{5} - 8\sqrt{4}\sqrt{5} = 3\cdot 3\sqrt{5} - 8\cdot 2\sqrt{5}$

$= 9\sqrt{5} - 16\sqrt{5} = -7\sqrt{5}$

ex: $\sqrt{2} + 6\sqrt{2} = 7\sqrt{2}$

TRY 21) $3\sqrt[3]{16} + \sqrt[3]{54}$

$= 3\sqrt[3]{8}\sqrt[3]{2} + \sqrt[3]{27}\sqrt[3]{2}$

$= 3\cdot 2\sqrt[3]{2} + 3\sqrt[3]{2} = 6\sqrt[3]{2} + 3\sqrt[3]{2} = 9\sqrt[3]{2}$

23) $\sqrt{a} + 3\sqrt{16a^3}$

$= \sqrt{a} + 3\sqrt{16}\sqrt{a^2}\sqrt{a} = \sqrt{a} + 3\cdot 4\cdot a\sqrt{a}$

$= \sqrt{a} + 12a\sqrt{a}$

$= (1 + 12a)\sqrt{a}$

ex [monomial · monomial]

$\sqrt{2}\cdot\sqrt{3} = \sqrt{6}$ $\qquad$ $\sqrt{2}\sqrt{8} = \sqrt{16} = 4$

$\sqrt[3]{4}\sqrt[3]{4} = \sqrt[3]{16} = \sqrt[3]{8}\sqrt[3]{2} = 2\sqrt[3]{2}$

$3\sqrt{5}\cdot 2\sqrt{7} = 6\sqrt{35}$ $\qquad$ $2\sqrt{3}\cdot 5\sqrt{3} = 2\cdot 5\cdot \sqrt{3}^2$

$= 10\cdot 3 = 30$

ex: [monomial · binomial]

$2\sqrt{5}(2\sqrt{2} + 4\sqrt{5}) = 2\sqrt{5}\cdot 2\sqrt{2} + 2\sqrt{5}\cdot 4\sqrt{5}$

$= 4\sqrt{10} + 40$

ex: $\sqrt{2}(5\sqrt{2} + 7\sqrt{3} + 6\sqrt{5}) = \sqrt{2}\cdot 5\sqrt{2} + \sqrt{2}\cdot 7\sqrt{3} + \sqrt{2}\cdot 6\sqrt{5}$

$= 10 + 7\sqrt{6} + 6\sqrt{10}$

ex: $\left(\sqrt{2}+\sqrt{3}\right)\left(\sqrt{5}+\sqrt{7}\right)$

$= \sqrt{2}\cdot\sqrt{5} + \sqrt{2}\cdot\sqrt{7} + \sqrt{3}\cdot\sqrt{5} + \sqrt{3}\cdot\sqrt{7}$

$= \sqrt{10} + \sqrt{14} + \sqrt{15} + \sqrt{21}$

ex: $\left(\sqrt{2}+\sqrt{3}\right)^2 = \left(\sqrt{2}+\sqrt{3}\right)\left(\sqrt{2}+\sqrt{3}\right)$

$= \sqrt{2}^2 + \sqrt{2}\cdot\sqrt{3} + \sqrt{2}\cdot\sqrt{3} + \sqrt{3}^2$

$= \sqrt{2}^2 + 2\sqrt{2}\sqrt{3} + \sqrt{3}^2$

$= 2 + 2\sqrt{6} + 3$

$= 5 + 2\sqrt{6}$

FACTS:

1. $(a+b)^2 = a^2 + 2ab + b^2$
2. $(a-b)^2 = a^2 - 2ab + b^2$

} perfect square trinomial

3. $(a-b)(a+b) = a^2 - b^2$

difference of squares

ex: $\left(2\sqrt{5} - 3\sqrt{3}\right)^2 = \left(2\sqrt{5}\right)^2 - 2\left(2\sqrt{5}\right)\left(3\sqrt{3}\right) + \left(3\sqrt{3}\right)^2$

$= 20 - 12\sqrt{15} + 27$

$= 47 - 12\sqrt{15}$ (or $-12\sqrt{15} + 47$)

ex: $\left(2\sqrt{5} - \sqrt{3}\right)\left(2\sqrt{5} + \sqrt{3}\right) = \left(2\sqrt{5}\right)^2 - \left(\sqrt{3}\right)^2$

$= 20 - 3 = 17$

Rationalizing (binomial) denominators

ex: $$\frac{1}{\sqrt{7}-\sqrt{5}} \cdot \frac{\sqrt{7}+\sqrt{5}}{\sqrt{7}+\sqrt{5}} = \frac{\sqrt{7}+\sqrt{5}}{\sqrt{7}^2-\sqrt{5}^2}$$

$$= \frac{\sqrt{7}+\sqrt{5}}{7-5} = \frac{\sqrt{7}+\sqrt{5}}{2}$$

64) $$\frac{1+\sqrt{2}}{3+\sqrt{5}} \cdot \frac{3-\sqrt{5}}{3-\sqrt{5}} = \frac{3-\sqrt{5}+3\sqrt{2}-\sqrt{2}\sqrt{5}}{3^2-\sqrt{5}^2}$$

$$= \frac{3-\sqrt{5}+3\sqrt{2}-\sqrt{10}}{9-5} = \frac{3-\sqrt{5}+3\sqrt{2}-\sqrt{10}}{4}$$

68) $$\frac{\sqrt{7}+\sqrt{5}}{\sqrt{5}+\sqrt{2}} \cdot \frac{\sqrt{5}-\sqrt{2}}{\sqrt{5}-\sqrt{2}} = \frac{\sqrt{35}-\sqrt{14}+5-\sqrt{10}}{\underbrace{\sqrt{5}^2-\sqrt{2}^2}_{5-2}}$$

$$= \frac{\sqrt{35}-\sqrt{14}+5-\sqrt{10}}{3}$$

70) $$\frac{5\sqrt{3}-\sqrt{11}}{2\sqrt{3}-5\sqrt{2}} \cdot \frac{2\sqrt{3}+5\sqrt{2}}{2\sqrt{3}+5\sqrt{2}} = \frac{30+25\sqrt{6}-2\sqrt{33}-5\sqrt{22}}{(2\sqrt{3})^2-(5\sqrt{2})^2}$$

$$= \frac{30+25\sqrt{6}-2\sqrt{33}-5\sqrt{22}}{12-50} = \frac{30+25\sqrt{6}-2\sqrt{33}-5\sqrt{22}}{-38}$$