Simulated Quiz/warm-up

(1) Add

$$3\sqrt{50} + 4\sqrt{8} + \sqrt{98}$$

$$= 3\sqrt{25}\sqrt{2} + 4\sqrt{4}\sqrt{2} + \sqrt{49}\sqrt{2}$$

$$= 3\cdot 5\sqrt{2} + 4\cdot 2\sqrt{2} + 7\sqrt{2}$$

$$= 15\sqrt{2} + 8\sqrt{2} + 7\sqrt{2} = 30\sqrt{2}$$

(2) Multiply

$$\sqrt{5}\left(2\sqrt{3} + 3\sqrt{5} + \sqrt{7}\right)$$

$$= \sqrt{5}\cdot 2\sqrt{3} + \sqrt{5}\cdot 3\sqrt{5} + \sqrt{5}\cdot\sqrt{7}$$

$$= 2\sqrt{15} + 15 + \sqrt{35}$$

(3) Rationalize the denominator

$$\frac{1}{3\sqrt{7}-2}\cdot\frac{3\sqrt{7}+2}{3\sqrt{7}+2} = \frac{3\sqrt{7}+2}{(3\sqrt{7})^2-2^2} = \frac{3\sqrt{7}+2}{63-4}$$

$$= \frac{3\sqrt{7}+2}{59}$$

(4)

$$\frac{\sqrt{7}+\sqrt{2}}{\sqrt{7}-\sqrt{2}}\cdot\frac{\sqrt{7}+\sqrt{2}}{\sqrt{7}+\sqrt{2}} = \frac{(\sqrt{7})^2 + 2(\sqrt{2})(\sqrt{7}) + (\sqrt{2})^2}{(\sqrt{7})^2-(\sqrt{2})^2}$$

$$= \frac{7 + 2\sqrt{14} + 2}{7-2} = \frac{9 + 2\sqrt{14}}{5}$$

or $\dfrac{2\sqrt{14}+9}{5}$

10.6 Solving Radical Equations

ex: $\sqrt{x} = 3$

$\sqrt{x}^2 = 3^2$

$x = 9$

ex: $\sqrt{x} = -7$

This is a positive number

This a negative number.

[The radical refers to the positive version of two square roots.]

$(\sqrt{x})^2 = (-7)^2$

$x = 49$ ← An extraneous solution.

Note: $3 = 3$ so $3^2 = 3^2 = 9$

$-5 = -5$ so $(-5)^2 = (-5)^2 = 25$

$7 \neq -7$ BUT $(7)^2 = (-7)^2 = 49$

ex: $\sqrt{x-2} = 5$

$(\sqrt{x-2})^2 = 5^2$ ← FLAG: This step may introduce extraneous solutions. So we must check our answer.

$x - 2 = 25$

$x = 27$ ← If there is a solution, it is 27.

check: $\sqrt{(27)-2} = \sqrt{25} \stackrel{\checkmark}{=} 5$

ex: Solve $\sqrt[3]{x+5} = 3$

$$\left(\sqrt[3]{x+5}\right)^3 = 3^3$$

$$x+5 = 27$$

$$x = 22 \quad \text{(and check)}$$

ex: 12) $\sqrt{2x} - 1 = 2$

$$+1 \qquad +1$$

$$\sqrt{2x} = 3$$

$$\left(\sqrt{2x}\right)^2 = 3^2 \quad \leftarrow \text{FLAG}$$

$$2x = 9$$

$$\boxed{x = \frac{9}{2}}$$

Check:

$\cancel{\sqrt{2\left(\frac{9}{2}\right)} = \sqrt{9}}$

$\sqrt{2\left(\frac{9}{2}\right)} - 1 = \sqrt{9} - 1$

$= 3 - 1 \overset{\checkmark}{=} 2$

ex: $x = \sqrt{x+10} + 10$

Isolate the radical $\rightarrow$ $x - 10 = \sqrt{x+10}$

$$(x-10)^2 = \sqrt{x+10}^{\,2} \quad \leftarrow \text{FLAG}$$

$$x^2 - 20x + 100 = x + 10$$

$$-x \quad -10 \qquad -x \quad -10$$

$$x^2 - 21x + 90 = 0$$

$$(x-6)(x-15) = 0$$

$$x - 6 = 0 \quad \text{or} \quad x - 15 = 0$$

$$\cancel{x = 6} \quad \text{or} \quad \boxed{x = 15}$$

extraneous

check: $x=6$

$6 \overset{?}{=} \sqrt{(6)+10} + 10$

$6 \neq 14$

Check: $x=15$

$15 \overset{?}{=} \sqrt{(15)+10} + 10$

$\overset{\checkmark}{=} \sqrt{25} + 10$

$= 5 + 10$

10.6 (cont'd) Two radicals in an equation

Method: Isolate one radical, square both sides, repeat.

ex: $\sqrt{x-2} + 2 = \sqrt{2x+3}$

$\left(\sqrt{x-2} + 2\right)^2 = \sqrt{2x+3}^{\,2}$ ← FLAG

$\sqrt{x-2}^{\,2} + 2\cdot 2\sqrt{x-2} + 2^2 = 2x+3$

$x-2+4\sqrt{x-2}+4 = 2x+3$

$4\sqrt{x-2} + x + 2 = 2x+3$ Isolate the latest radical.

$4\sqrt{x-2} = x+1$

$\left(4\sqrt{x-2}\right)^2 = (x+1)^2$

$4^2 \cdot \left(\sqrt{x-2}\right)^2 = x^2+2x+1$

$16(x-2) = x^2+2x+1$

$16x-32 = x^2+2x+1$

$0 = x^2-14x+33$

$0 = (x-11)(x-3)$

$x-11=0$ or $x-3=0$

$\boxed{x=11}$ or $\boxed{x=3}$

check: $x=11$

$\sqrt{11-2} + 2 \stackrel{?}{=} \sqrt{2(11)+3}$

$\sqrt{9} + 2 = \sqrt{25}$ ✓

$3+2=5$

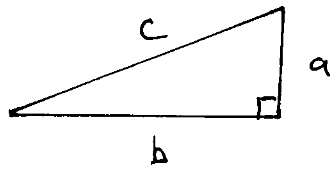
Check: $x=3$

$\sqrt{3-2} + 2 \stackrel{?}{=} \sqrt{2(3)+3}$

$\sqrt{1} + 2 = \sqrt{9}$ ✓

$1+2=3$

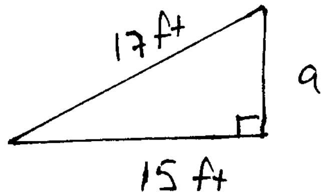
10.7 Pythagorean theorem and related topics



$$a^2+b^2=c^2$$

If you know two of a, b, or c, you can calculate the third.

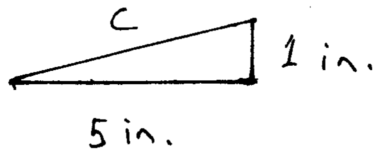
ex:



what is a?

$$a^2+15^2=17^2$$
$$a^2+225=289$$
$$a^2=64$$
$$\boxed{a=8\text{ ft}}$$

ex:



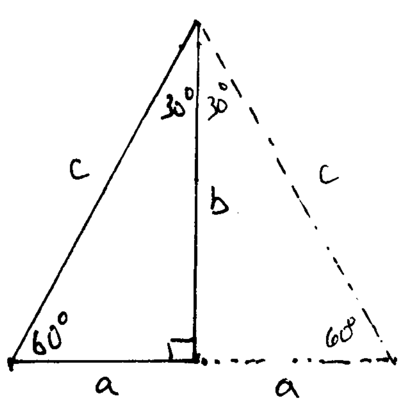
what is the hypotenuse?

$$1^2+5^2=c^2$$
$$1+25=c^2$$
$$26=c^2$$
$$c=\sqrt{26}\text{ inches} \simeq 5.10 \text{ inches}$$

$30^\circ-60^\circ-90^\circ$ Triangles

Use this together with the Pythagoren Theorem:

Aha!
$c=2a$



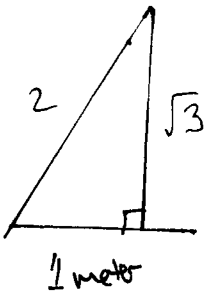
$$a^2+b^2=c^2$$
$$a^2+b^2=(2a)^2$$
$$a^2+b^2=4a^2$$
$$b^2=3a^2$$
$$\sqrt{b^2}=\sqrt{3a^2}$$
$$b=\sqrt{3}\cdot a$$

So for 30°-60°-90° triangles

$$\boxed{\begin{array}{c} b=a\sqrt{3} \\ c=2a \end{array}}$$

So if we know any one of a, b, or c, you know all three.

ex: If the shortest side of a $30°-60°-90°$ triangle is 1 meter, what are the lengths of the other sides?

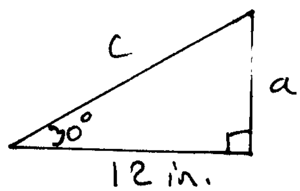


$a = 1$ meter

So $c = 2$ meters

and $b = \sqrt{3} = 1.732$

ex:



The base of this triangle is 12 inches. What are the other dimensions?

$12 = a\sqrt{3} \Rightarrow a = \frac{12}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{12\sqrt{3}}{3}$

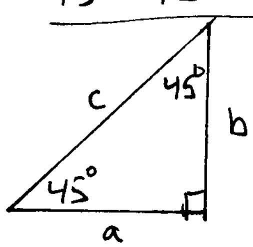
$= 4\sqrt{3}$ inches

≈ 6.93 inches

and $c = 2a = 2(4\sqrt{3}) = 8\sqrt{3}$

≈ 13.86 inches

<u>$45°-45°-90°$ triangles</u> or "isosceles right triangle"



Aha! $a = b$

Combine with Pythagorean thm

$a^2 + b^2 = c^2$

$a^2 + a^2 = c^2$

$2a^2 = c^2$

$\sqrt{2a^2} = \sqrt{c^2}$

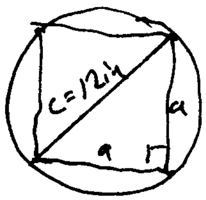
$a\sqrt{2} = c$

So for isosceles right triangles

$$\boxed{\begin{array}{l} a = b \\ c = a\sqrt{2} \end{array}}$$

So if we know any <u>one</u> of a, b, or c, we know all three.

ex: You have a square peg which barely fits in a round hole of diameter 12 inches. What are the lengths of the sides of the peg?



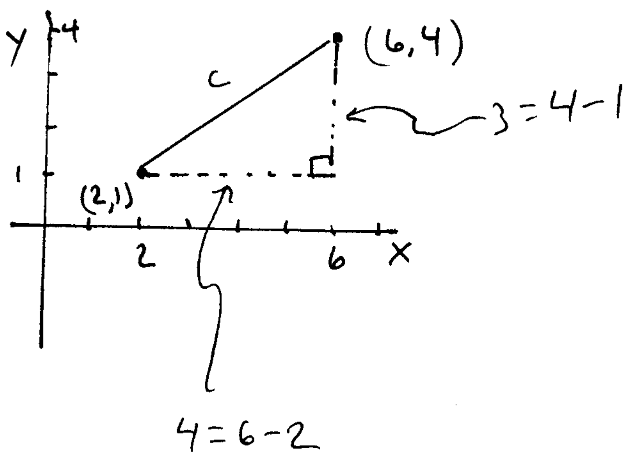
$a = b$

$c = a\sqrt{2}$

$12 = a\sqrt{2} \Rightarrow a = \frac{12}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{12\sqrt{2}}{2} = 6\sqrt{2}$ inches

≈ 8.49 inches

Idea the distance formula



How far apart are (2,1) and (6,4)?

Equivalent: what's the length of the line segment connecting the two points?

$$4^2 + 3^2 = c^2$$

$$(6-2)^2 + (4-1)^2 = c^2$$

or $c = \sqrt{(6-2)^2 + (4-1)^2} = \sqrt{4^2 + 3^2}$

$= \sqrt{16 + 9}$

$= \sqrt{25}$

$= 5$