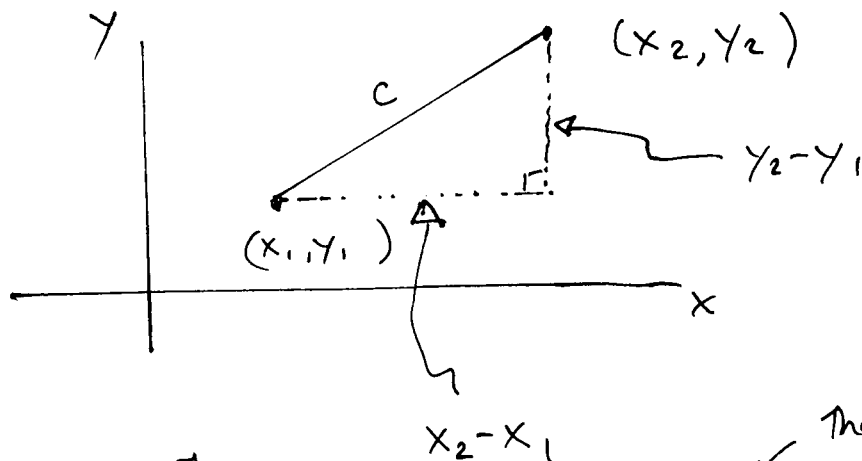


10.7 (cont'd) How to derive the distance formula

What is the distance between these two points?



By the Pythagorean Theorem:

$$c^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$$

That is,
 $c^2 = a^2 + b^2$
 with $a = x_2 - x_1$
 $b = y_2 - y_1$

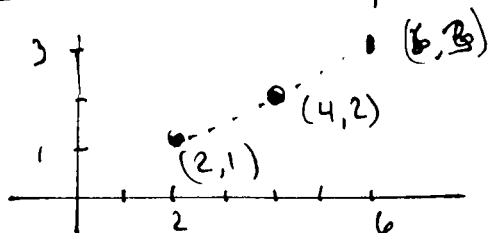
distance
between
 (x_1, y_1) and
 (x_2, y_2)

 $= c = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

ex: Find the distance between $(-6, 4)$ and $(-1, 5)$.

$$\begin{aligned} \text{distance} &= \sqrt{[-6 - (-1)]^2 + [4 - 5]^2} \\ &= \sqrt{(-5)^2 + (-1)^2} = \sqrt{25 + 1} = \sqrt{26} \\ &\approx 5.099 \end{aligned}$$

Midpt: Find the midpt between $(2, 1)$ and $(6, 3)$.

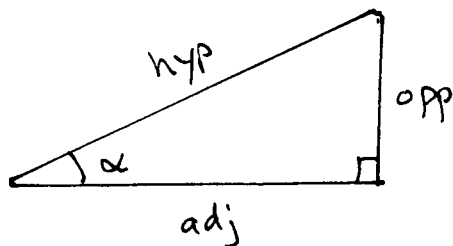


$$\frac{2+6}{2} = 4$$

$$\frac{1+3}{2} = 2$$

Intro to trigonometry

Definition: SOH CAH TOA



$$\sin \alpha = \frac{\text{opp}}{\text{hyp}}$$

$$\cos \alpha = \frac{\text{adj}}{\text{hyp}}$$

$$\tan \alpha = \frac{\text{opp}}{\text{adj}}$$

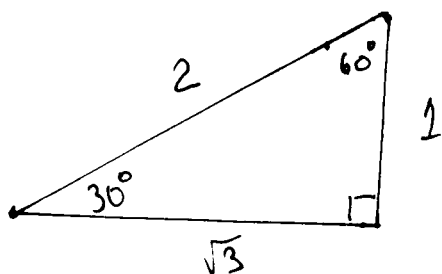
ex Estimate, by measuring carefully,

$$\sin 25^\circ = \frac{\text{opp}}{\text{hyp}} = \frac{7/8}{2+1/8} = \frac{.875}{2.125} \approx .412 \stackrel{\text{exact}}{=} .423$$

$$\cos 25^\circ = \frac{\text{adj}}{\text{hyp}} = \frac{1+15/16}{2+1/8} \approx \frac{1.9375}{2.125} \approx .912 = .906$$

$$\tan 25^\circ = \frac{\text{opp}}{\text{adj}} = \frac{.875}{1.9375} \approx .452 = .466$$

ex: without measuring, find sine, cosine and tangent of 30° .



$$\sin 30^\circ = \frac{\text{opp}}{\text{hyp}} = \frac{1}{2} = .500$$

$$\cos 30^\circ = \frac{\text{adj}}{\text{hyp}} = \frac{\sqrt{3}}{2} \approx .866$$

$$\tan 30^\circ = \frac{\text{opp}}{\text{adj}} = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3} \approx .577$$

$$\sin 60^\circ = \frac{\text{opp}}{\text{hyp}} = \frac{\sqrt{3}}{2} = .866$$

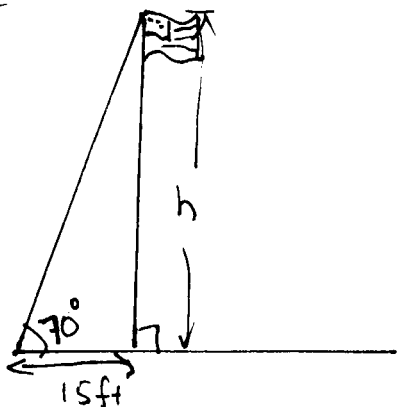
$$\cos 60^\circ = \frac{\text{adj}}{\text{hyp}} = \frac{1}{2} = .500$$

$$\tan 60^\circ = \frac{\text{opp}}{\text{adj}} = \frac{\sqrt{3}}{1} = \sqrt{3} = 1.732$$

Solving right triangles

Fact: Roughly speaking, if you know 3 of the 6 quantities of a triangle, you can calculate the other three.

ex



How tall is this flagpole, to the nearest tenth of a foot?

$$\tan 70^\circ = \frac{\text{opp}}{\text{adj}}$$

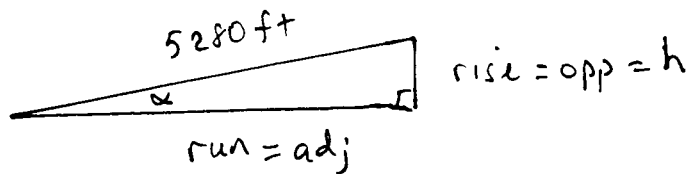
$$2.747 = \frac{h}{15 \text{ ft}}$$

$$h = 15 (\tan 70^\circ) = 15 (2.747) \approx 41.2 \text{ ft}$$

ex: You drive ^{uphill} on a 13% grade. After your odometer has gone up exactly 1.0 miles (= 5280 feet), how much elevation have you gained?

$$0.13 = 13\% = \frac{13}{100} = \frac{\text{rise}}{\text{run}} = \tan \alpha$$

What is the angle α ?



$$\tan \alpha = 0.13$$

$$\begin{aligned} \alpha &= \tan^{-1} 0.13 \\ &= \arctan 0.13 \\ &= \text{angle whose tangent is } 0.13 \\ &= 7.40^\circ \end{aligned}$$

$$\sin 7.40^\circ = \frac{\text{opp}}{\text{hyp}}$$

$$\begin{aligned} \sin 7.40^\circ &= \frac{h}{5280} \Rightarrow h = 5280 (\sin 7.40^\circ) \\ &= 5280 (.129) \\ &= 681 \text{ feet} \end{aligned}$$

Remark. We will talk more about arcsine, arccosine, and arctangent, later.