11.3 Going *from* the solution *to* the quadratic equ

example: Find a quadratic equation with solutions

$x = \frac{3}{2}$ or $x = -\frac{5}{6}$

Idea: think backwards (i.e "solving by factoring" backwards)

$2x = 3$ or $6x = -5$

$2x - 3 = 0$ OR $6x + 5 = 0$

$$(2x-3)(6x+5) = 0$$

$$12x^2 + 10x - 18x - 15 = 0$$

$$12x^2 - 8x - 15 = 0$$

TRY THIS: Find a quadratic equation with solution set $= \{-4, \frac{2}{3}\}$

$x = -4$ OR $x = \frac{2}{3}$

$x + 4 = 0$ OR $3x = 2$

OR $3x - 2 = 0$

$$(x+4)(3x-2) = 0$$

$$3x^2 - 2x + 12x - 8 = 0$$

$$3x^2 + 10x - 8 = 0$$

[After break: Solutions are $x = 2 \pm \sqrt{3}$.] ← NOT IN THE TEXTBOOK.

[NOT IN TEXTBOOK]

ex: Find a quadratic equation with solutions

6 $x = 2 \pm \sqrt{3}$ [Idea: "solving by completing the square" backwards]

5 $x - 2 = \pm\sqrt{3}$

4 $(x-2)^2 = (\pm\sqrt{3})^2 = 3$

$x^2 - 4x + 4 = 3$

1 and 3 $x^2 - 4x + 1 = 0$

Try this: Find a quadratic equation with solutions

$x = 3 \pm i$ equivalent: $x = 3 \pm \sqrt{-1}$

$x - 3 = \pm i$

$(x-3)^2 = (\pm i)^2 = i^2 = -1$

$x^2 - 6x + 9 = -1$

$x^2 - 6x + 10 = 0$

ex: Find a quadratic eqn with solutions $x = \pm 7i$

$x^2 = (\pm 7i)^2 = (\pm 1)^2 \cdot (7)^2 \cdot (i)^2$

$x^2 = 1 \cdot 49 \cdot (-1) = -49$

$x^2 + 49 = 0$

11.3 Discriminant $= b^2-4ac$

ex: Given $3x^2-11x+8=0$. without solving,

answer: 1 Are the solutions real?

2 If so, are the solutions rational?

Equivalent: That is, could we solve by factoring?

$$b^2-4ac = (-11)^2-4(3)(8)$$

$$= 121-96 = 25$$

answer to 1: YES because $25 = b^2-4ac > 0$

answer to 2: YES because $25 = 5^2$ so that $\sqrt{25} = \sqrt{b^2-4ac} = 5$.

what are the solutions? $x = \dfrac{-(-11) \pm \sqrt{25}}{2(3)}$

$$= \frac{11 \pm 5}{6} = \begin{cases} \frac{11+5}{6} = \frac{16}{6} = \frac{8}{3} \text{ OR} \\ \frac{11-5}{6} = \frac{6}{6} = 1 \end{cases}$$

Solve by factoring:

$ac = 3 \cdot 8 = 24$

p, q	p+q
-3, -8	-11

$$3x^2-11x+8=0$$

$$3x^2-3x-8x+8=0$$

$$3x(x-1)-8(x-1)=0$$

$$(3x-8)(x-1)=0$$

$$\updownarrow \qquad \updownarrow$$

$$8/3 \qquad 1$$

ex: $x^2 + 6x + 34 = 0$

Q: Are the solutions REAL or COMPLEX?

$b^2 - 4ac = 6^2 - 4(1)(34) = 36 - 136 = -100 < 0$

$\therefore$ The solutions are NOT REAL, because $b^2-4ac = -100 < 0$.

What are the solutions?

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{-6 \pm \sqrt{-100}}{2}$$

$$= \frac{-6 \pm 10i}{2} = \frac{-6}{2} \pm \frac{10i}{2}$$

$$= -3 \pm 5i$$