11.4 Applications involving quadratic equations

ex: $A = \pi r^2$ Solve this equation for r.

$$\frac{A}{\pi} = \frac{\pi r^2}{\pi}$$

$$\sqrt{\frac{A}{\pi}} = \sqrt{r^2}$$

$$r = \sqrt{\frac{A}{\pi}}$$

13) Solve for b:

$$a^2 + b^2 = c^2$$
$$-a^2 \qquad -a^2$$
$$b^2 = c^2 - a^2$$
$$\sqrt{b^2} = \sqrt{c^2 - a^2}$$
$$b = \sqrt{c^2 - a^2}$$

17) $N = \frac{1}{2}(n^2 - n)$ Solve n.

A Quadratic equation where n (not x) is the variable and N is a constant.

$$2N = n^2 - n$$

$$0 = n^2 - n - 2N$$ $a = 1$, $b = -1$, $c = -2N$

number of teams (or handshakes) $= n = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \dfrac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(-2N)}}{2(1)}$

$$n = \frac{1 \pm \sqrt{1 + 8N}}{2}$$ Take the '+' case only

$$n = \frac{1 + \sqrt{1+8N}}{2}$$ where N = number of games (or handshakes)

17 cont'd) How we could use this.

Suppose there are 10 games (or 10 handshakes), how many teams (or handshakers) are there?

$$n = \frac{1+\sqrt{1+8N}}{2} \quad \text{and take } N=10,$$

to get $n = \dfrac{1+\sqrt{1+8(10)}}{2} = \dfrac{1+\sqrt{81}}{2} = \dfrac{1+9}{2} = \dfrac{10}{2} = 5$

11.4 33) $\xrightarrow[\text{fast}]{120 \text{ miles}}$ $\xrightarrow[\text{10 mph slower}]{100 \text{ miles}}$

total time = 4 hours.
Speed on each portion of trip? Use $d = r \cdot t$,

So $\dfrac{d}{r} = t$

	distance =	rate ·	time
1st portion	120 miles	r mph	$\frac{120}{r}$ hours
2nd portion	100 mi	$r-10$	$\frac{100}{r-10}$ hrs

Equation: $\dfrac{120}{r} + \dfrac{100}{r-10} = 4$ Multiply by the LCD: $r(r-10)$:

$$\frac{120\cancel{r}(r-10)}{\cancel{r}} + \frac{100r\cancel{(r-10)}}{\cancel{r-10}} = 4r(r-10)$$

$$120r - 1200 + 100r = 4r^2 - 40r$$

$$220r - 1200 = 4r^2 - 40r$$

$$0 = 4r^2 - 40r - 220r + 1200$$

$$0 = 4r^2 - 260r + 1200$$

Divide by 4:

$$\frac{0}{4} = \frac{4r^2}{4} - \frac{260r}{4} + \frac{1200}{4}$$

$$0 = r^2 - 65r + 300$$

$$0 = (r-60)(r-5)$$

$r - 60 = 0$ or $r - 5 = 0$

$\boxed{\begin{array}{l} r = 60 \text{ mph} \\ r - 10 = 50 \text{ mph} \end{array}}$ or $\cancel{r = 5}$ mph

Throw this out, for $r - 10 = -5$ mph

Check: 1^{st} time $= \frac{120}{r} = \frac{120}{60} = 2$ hrs

2^{nd} time $= \frac{100}{r-10} = \frac{100}{50} = 2$ hrs

4 hrs

11.5 Equations Reducible to Quadratic

ex: $x^4 - 8x^2 - 9 = 0$

Is this a quadratic equation? No, this is a "quartic equation."

$$(x^2)^2 - 8x^2 - 9 = 0$$

Make the following substitution: $u = x^2$ so $u^2 = (x^2)^2 = x^4$

$$u^2 - 8u - 9 = 0$$

$$(u-9)(u+1) = 0$$

$u - 9 = 0$ or $u + 1 = 0$

$u = 9$ or $u = -1$

Back-Sub x: $x^2 = 9$ or $x^2 = -1$

$x = \pm\sqrt{9} = \pm 3$ OR $x^2 = \pm\sqrt{-1} = \pm i$ ← Total of four solutions

33) $t^{2/3} + t^{1/3} - 6 = 0$

Observe: $t^{2/3} = (t^{1/3})^2$ so Let $u = t^{1/3}$

so $u^2 = (t^{1/3})^2 = t^{2/3}$

Step 1: Solving the quadratic eqn. in u

$$u^2 + u - 6 = 0$$

$$(u+3)(u-2) = 0$$

$$u+3=0 \quad \text{or} \quad u-2=0$$

$$u=-3 \quad \text{or} \quad u=2$$

Step 2: Solve the resulting equations

$$t^{1/3} = -3 \quad \text{or} \quad t^{1/3} = 2$$

$$\sqrt[3]{t} = -3 \quad \text{or} \quad \sqrt[3]{t} = 2$$

$$(\sqrt[3]{t})^3 = (-3)^3 \quad \text{or} \quad (\sqrt[3]{t})^3 = 2^3$$

$$t = -27 \quad \text{or} \quad t = 8$$

12) $x^4 - 17x^2 + 16 = 0$ Let $u = x^2$

$$u^2 - 17u + 16 = 0$$

$$(u-1)(u-16) = 0$$

$$u-1=0 \quad \text{or} \quad u-16=0$$

$$u=1 \quad \text{or} \quad u=16$$

$$x^2=1 \quad \text{or} \quad x^2=16$$

$$x = \pm\sqrt{1} = \pm 1 \quad \text{or} \quad x = \pm\sqrt{16} = \pm 4$$

34) $w^{2/3} - 2w^{1/3} - 8 = 0$ Let $u = w^{1/3}$

$u^2 - 2u - 8 = 0$

$(u \quad 4)(u+2) = 0$

$u - 4 = 0$ or $u + 2 = 0$

$u = 4$ or $u = -2$

$w^{1/3} = 4$ or $w^{1/3} = -2$

$(w^{1/3})^3 = 4^3$ or $(w^{1/3})^3 = (-2)^3$

$w = 64$ or $w = -8$