Warm-up / Mid-Chapter Review (p.733)

3) $x^2 + 4x = 21$ OR solve by factoring

$x^2 + 4x + 4 = 21 + 4$

$(x+2)^2 = 25$

$x + 2 = \pm\sqrt{25} = \pm 5$

$x = -2 \pm 5 = \begin{cases} 3 \\ -7 \end{cases}$ OR

4) $t^2 - 196 = 0$

$t^2 = 196$

$t = \pm\sqrt{196} = \pm 14$

5) $x^2 = 2x + 5$ OR use the quadratic formula

$x^2 - 2x = 5$ Solve $x^2 - 2x - 5 = 0$

$x^2 - 2x + 1 = 5 + 1$ $a = 1 \quad b = -2 \quad c = -5$

$(x-1)^2 = 6$

$x - 1 = \pm\sqrt{6}$

$x = 1 \pm \sqrt{6}$

6) $x^2 = 2x - 5$

$x^2 - 2x + 1 = -5 + 1$

$(x-1)^2 = -4$

$x - 1 = \pm\sqrt{-4} = \pm 2i$

$x = 1 \pm 2i$

OR use the Quadratic formula

$x^2 - 2x + 5 = 0$

$a = 1 \quad b = -2 \quad c = 5$

7) $x^4 = 16$ [see 11.5]

$(x^2)^2 = 16$ and let $u = x^2$

$u^2 = 16$

$u = \pm\sqrt{16} = \pm 4$

$u = 4$ or $u = -4$

$x^2 = 4$ OR $x^2 = -4$

$x = \pm\sqrt{4}$ OR $x = \pm\sqrt{-4}$

$= \pm 2$ $= \pm 2i$

NOTE: A degree 4 polynomial equation has 4 solutions.

8) $(t+3)^2 = 7$

$t+3 = \pm\sqrt{7}$

$t = -3 \pm\sqrt{7}$

Remark: You could multiply out then use the quadratic formula, but that's not wise.

9) $n(n-3) = 2n(n+1)$

$n^2 - 3n = 2n^2 + 2n$

$-n^2 + 3n \quad -n^2 + 3n$

$0 = n^2 + 5n$

$0 = n(n+5)$

$n = 0$ or $n+5 = 0$

$n = -5$

10) $6y^2 - 7y - 10 = 0$

Hmmm. I wonder if we can solve this by factoring.

Can we factor this? Equivalent:

Is $b^2 - 4ac$ a perfect square?

$$b^2 - 4ac = (-7)^2 - 4(6)(-10)$$

$$= 49 + 240 = 289 = 17^2$$

Yes! To both questions!

Quadratic formula: $\cancel{y} = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$= \frac{7 \pm \sqrt{17^2}}{2(6)} = \frac{7 \pm 17}{12}$$

$$= \begin{cases} \frac{24}{12} = 2 & \text{OR} \\ -\frac{10}{12} = -\frac{5}{6} \end{cases}$$

Can we check this by solving by factoring?

$$6y^2 - 7y - 10 = 0$$

$$(6y + 5)(y - 2) = 0$$

Solutions: $y = -\frac{5}{6}$ $\quad y = 2$

11) $16c^2 = 7c$

$16c^2 - 7c = 0$

$c(16c - 7) = 0$

$c = 0$ OR $16c - 7 = 0$

$16c = 7$

$c = \frac{7}{16}$

12) $3x^2 + 5x = 1$

$3x^2 + 5x - 1 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-5 \pm \sqrt{5^2 - 4(3)(-1)}}{2(3)}$$

$$= \frac{-5 \pm \sqrt{25 + 12}}{6} = \frac{-5 \pm \sqrt{37}}{6}$$

13) $(t+4)(t-3) = 18$

$t^2 - 3t + 4t - 12 = 18$

$t^2 + t - 30 = 0$

$(t-5)(t+6) = 0$

$t - 5 = 0$ or $t + 6 = 0$

$t = 5$ or $t = -6$

14) $(m^2+3)^2 - 4(m^2+3) - 5 = 0$

How many solutions will this equation have? 4.

Let $u = m^2+3$:

$$u^2 - 4u - 5 = 0$$

$$(u-5)(u+1) = 0$$

$u-5=0$ or $u+1=0$

$u=5$ or $u=-1$

$m^2+3=5$ or $m^2+3=-1$

$-3\ \ -3$ $\qquad\qquad$ $-3\ \ -3$

$m^2=2$ $\qquad\qquad$ $m^2=-4$

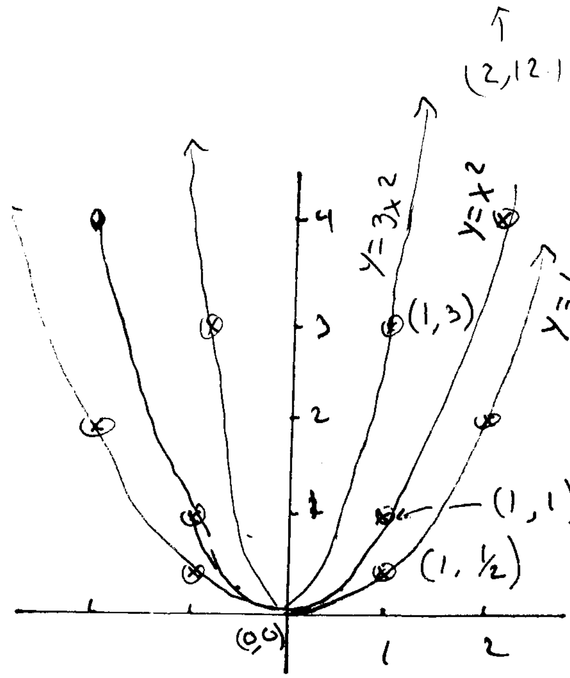
$m=\pm\sqrt{2}$ $\qquad\qquad$ $m=\pm\sqrt{-4}=\pm 2i$

11.6 Graphs of the form:

$$y = a(x-h)^2 + k$$

$$y = ax^2$$

x	x^2	$3x^2$	$\frac{1}{2}x^2$
-2	4	12	2
-1	1	3	1/2
0	0	0	0
1	1	3	1/2
2	4	12	2



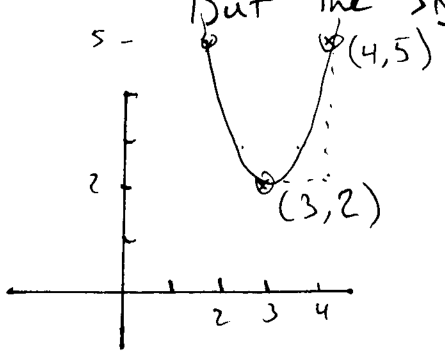
ex: $y = 3(x-3)^2 + 2$

the vertex is where $x-3=0$ so $x=3$

then $y = 3(3-3)^2 + 2 = 0 + 2 = 2$

so vertex : $(x,y) = (3,2)$.

But the shape of the parabola is the same as $y = 3x^2$.



"Vertex form" of a quadratic function

$$\boxed{y = a(x-h)^2 + k}$$

This equation will have, as its graph, a parabola.

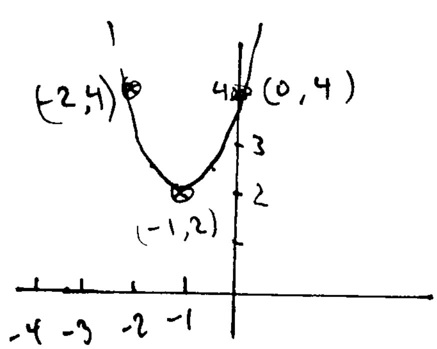
vertex = (h, k)

If $a > 0$, the parabola is open up.

Also, a determines the width of the parabola.

examples	vertex?	open up/down?
$y = (x-1)^2 + 2$	$(1, 2)$	up
$y = -2(x+3)^2 + 1$	$(-3, 1)$	down
$y = 5(x-7)^2$	$(7, 0)$	up
$y = 5x^2 - 7$	$(0, -7)$	up
$y = \frac{(x-2)^2}{3} = \frac{1}{3}(x-2)^2 + 0$	$(2, 0)$	up because $a = \frac{1}{3}$, $h = 2$, $k = 0$

ex: Here's a parabola. What is its equation?



$h = -1$
$k = 2$
$a = 2$

Equation:

$$y = 2(x+1)^2 + 2$$
$$= \underset{a}{2}\,[x - \underset{h}{(-1)}]^2 + \underset{k}{2}$$

x	y
-2	4
-1	2
0	4