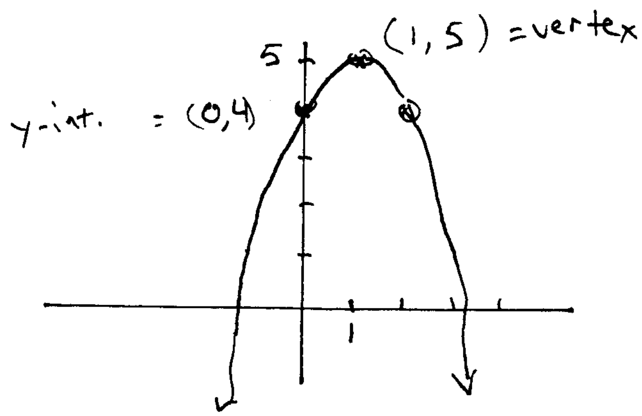
11.6 The graph of $y = a(x-h)^2 + k$ (continued)

ex: [NOT IN TEXTBOOK] Graph → Equation



Find the equation of the quadratic function in the form

$$y = a(x-h)^2 + k$$

for suitable choices of a, h, and k.

Given: $(h, k) = (1, 5)$ so our equation is

$$y = a(x-1)^2 + 5$$

So, what is a? Choose a so that $(x, y) = (0, 4)$ satisfies the equation: that is, this equation is true,

$$4 = a(0-1)^2 + 5$$

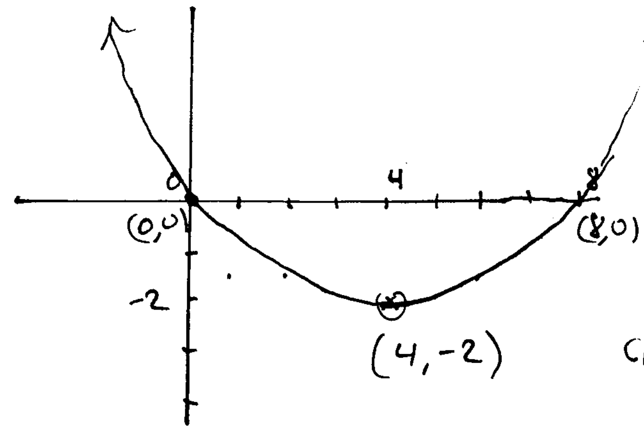
$$4 = a + 5 \Rightarrow a = -1$$

$\therefore$ $\boxed{y = -(x-1)^2 + 5}$ is the equation of the parabola.

partial check:

x	y
-1	1
0	4
1	5
2	4
3	1

ex:



Find an equation of the "vertex form" i.e.

$y = a(x-h)^2 + k$

Clear: $h = 4$

$k = -2$

$a = ?$ is positive

Use $(x,y) = (0,0)$ will satisfy the equation:

$$y = a(x-4)^2 - 2$$

$$0 = a(0-4)^2 - 2$$

$$0 = 16a - 2 \Rightarrow 2 = 16a \Rightarrow a = \frac{2}{16}$$

So $a = \frac{1}{8}$

$$\boxed{y = \frac{1}{8}(x-4)^2 - 2}$$

Check:

x	y
0	0
4	-2
8	$\frac{1}{8}(8-4)^2 - 2 = \frac{1}{8} \cdot 4^2 - 2 = \frac{16}{8} - 2 = 2 - 2 = 0$

11.7 The graph of $y = ax^2 + bx + c$

ex: [Fact: An eqn of the form $y = ax^2 + bx + c$
can be ~~re~~written as $y = a(x-h)^2 + k$]

$$y = x^2 + 8x + 3$$
$$y = x^2 + 8x + 16 + 3 - 16$$
$$y = (x+4)^2 - 13$$

so the vertex $= (-4, -13)$
open up, because $a = 1$.

ex: $y = 3x^2 + 12x - 7$

$a = 3$
$b = 12$
$c = -7$

$$= (3x^2 + 12x) - 7$$
$$= 3(x^2 + 4x \quad) - 7$$
$$= 3(x^2 + 4x + 4) - 7 - 12$$
$$= 3(x+2)^2 - 19$$

$a = 3$
$h = -2$
$k = -19$

so vertex $= (-2, -19)$
and $a = 3 > 0$ so open up

Fact: By studying this example

(shortcut) $f(x) = ax^2 + bx + c$ can written as

$f(x) = a(x-h)^2 + k$ where

$$\boxed{\begin{aligned} a &= a \\ h &= -\frac{b}{2a} \\ k &= f(h) \end{aligned}}$$

<u>ex</u>: $f(x) = -2x^2 + 8x + 3$

write as $f(x) = a(x-h)^2 + k$ (by shortcut method)

$$a = -2$$

$$h = -\frac{b}{2a} = -\frac{8}{2(-2)} = 2$$

$$k = f(2) = -2(2)^2 + 8(2) + 3$$

$$= -8 + 16 + 3 = 11$$

$$f(x) = -2(x-2)^2 + 11$$

<u>Remark</u>: $x = \underbrace{\boxed{\frac{-b}{2a}}}_{h} \pm \frac{\sqrt{b^2-4ac}}{2a}$

This means there is nothing new to memorize.