## Warm-up

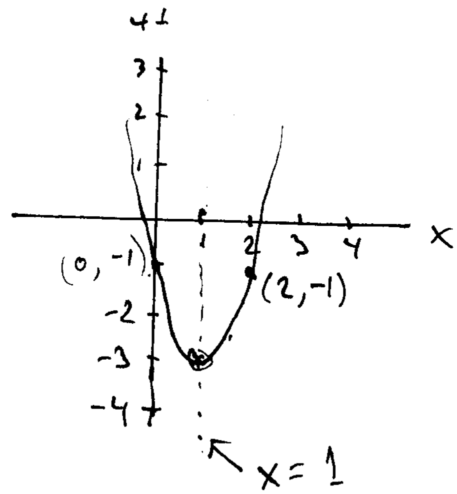
(1) for $f(x) = 2(x-1)^2 - 3$

a) what is the vertex? $(1,-3)$

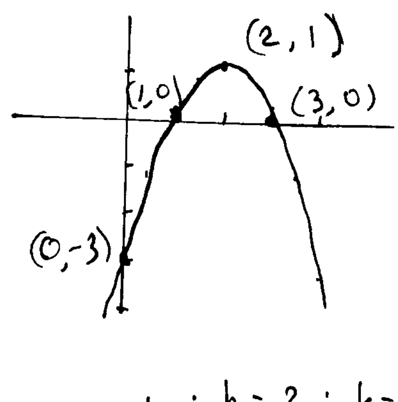
b) Open up or down? Open up because $a = 2 > 0$.

c)

| $x$ | $f(x)$ |
|---|---|
| 0 | $-1$ |
| vertex → 1 | $-3$ |
| 2 | $2(2-1)^2 - 3 = 2 - 3 = -1$ |




(2) Find an equation in the form ("vertex form"), $y = a(x-h)^2 + k$ for this parabola.




$y = a(x-2)^2 + 1$

To find $a$, use that $(x,y) = (3,0)$ satisfies the eqn.

$0 = a(3-2)^2 + 1$

$0 = a + 1$

$a = -1$

Answer: $a = -1$ ; $h = 2$ ; $k = +1$

$$\boxed{y = -(x-2)^2 + 1}$$

Remark: This is not taught in the textbook.

# 11.7 Graphs of $f(x) = ax^2 + bx + c$ ("general form" of a quadratic function)

19) For $\boxed{f(x) = x^2 + 4x + 5}$

a) Find the vertex, and axis of symmetry

b) Graph the function, find any x- and y-intercepts.

a) $a = 1 \qquad b = 4 \qquad c = 5$

$$h = -\frac{b}{2a} = -\frac{4}{2(1)} = -2$$

$$k = f(-2) = (-2)^2 + 4(-2) + 5$$
$$= 4 - 8 + 5 = 1$$

vertex $= (-2, 1)$

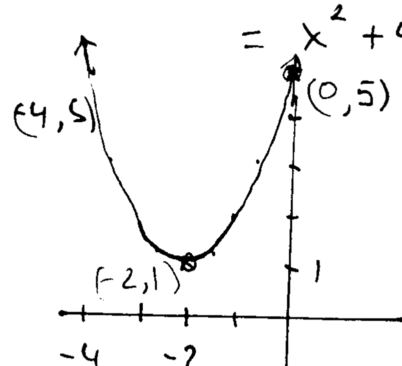
axis of symmetry: $x = -2$

Equation in "vertex form": $\boxed{f(x) = (x+2)^2 + 1}$

Check, by multiplying out: $f(x) = x^2 + 2 \cdot 2x + 2^2 + 1$
$= x^2 + 4x + 5$ ✓

b)

| | x | f(x) |
|---|---|---|
| | -3 | 2 |
| vertex → | -2 | 1 |
| | -1 | 2 |
| y-intercept → | 0 | 5 |




No x-intercepts

x-int's? Set $y = 0$, solve for x:

$0 = (x+2)^2 + 1 \Rightarrow (x+2)^2 = -1$

$\Rightarrow x + 2 = \pm\sqrt{-1} = \pm i$

$x = -2 \pm i$ ← Not real. So No x-ints.

## Finding intercepts

case 1: $f(x) = a(x-h)^2 + k$

ex: $f(x) = 3(x-1)^2 - 12$

[By the way, the vertex is $(1,-12)$]

x-intercepts? Set $y=0$ and solve for $x$:

$$3(x-1)^2 - 12 = 0$$

$$3(x-1)^2 = 12$$ Divide by 3:

$$(x-1)^2 = 4$$

$$x-1 = \pm\sqrt{4} = \pm 2$$

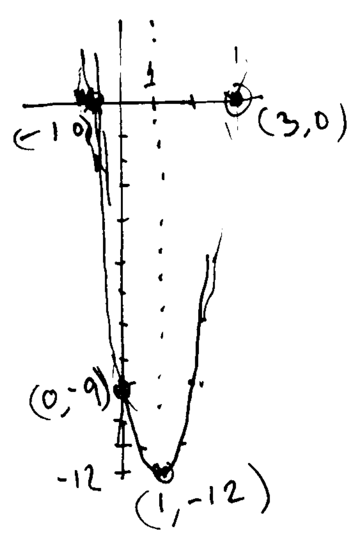
$$x = 1 \pm 2 = \begin{cases} 3 \\ -1 \end{cases}$$

So the x-intercepts are $(3,0)$ and $(-1,0)$

y-intercepts? Set $x=0$ then "solve" for $y$.

$$f(0) = 3(0-1)^2 - 12 = 3\cdot 1 - 12 = -9$$

| | $x$ | $f(x)$ |
|---|---|---|
| x-int → | -1 | 0 |
| y-int → | 0 | -9 |
| vertex → | 1 | -12 |
| x-int → | 3 | 0 |



Finding intercepts (cont'd)

case 2: $f(x) = ax^2 + bx + c$

ex: $f(x) = -x^2 + 2x + 8$

x-intercepts? Set $y=0$ then solve for $x$

$$-x^2+2x+8=0$$

$$\frac{-1\cdot(x^2-2x-8)}{-1} = \frac{0}{-1}$$

$$x^2-2x-8=0$$

$$(x-4)(x+2)=0$$

$x-4=0$ or $x+2=0$

$x=4$ or $x=-2$ $\Rightarrow$ $(4,0)$, $(-2,0)$ are the x-intercepts

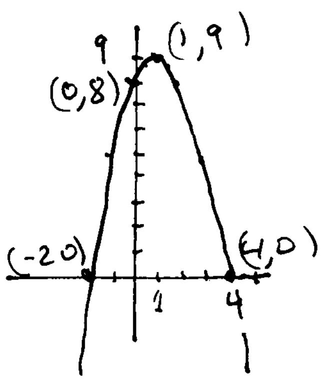
| | $x$ | $f(x)$ |
|---|---|---|
| x-int → | $-2$ | $0$ |
| y-int → | $0$ | $8$ |
| vertex → | $1$ | $9$ |
| x → int | $4$ | $0$ |

y-intercept? $f(0) = 8$ $\Rightarrow$ $(0,8)$ is the y-intercept.

Note: The axis of symmetry is midway between the two x-intercepts so $h = \frac{-2+4}{2} = 1$

and $f(1) = -(1^2)+2(1)+8 = -1+2+8 = 9$

that is, the vertex is $(1,9)$.



ex: what are the x- and y-intercepts of

$$f(x) = 3x^2 + 7x - 20$$

y-intercept? $(0, -20)$

x-intercepts? Solve

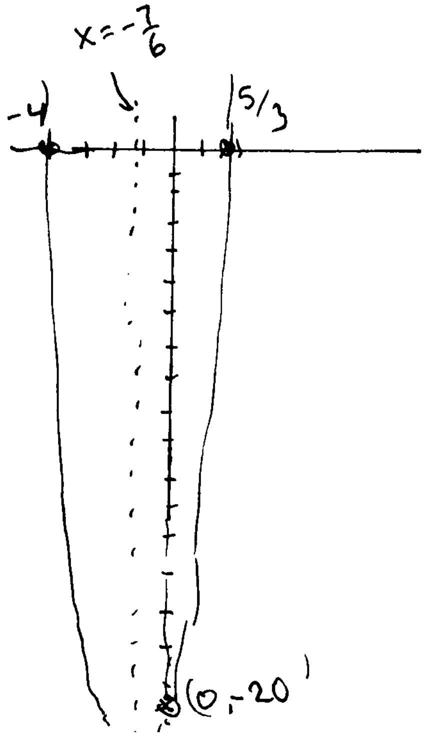
$$3x^2 + 7x - 20 = 0$$

$$x = \frac{-(7) \pm \sqrt{7^2 - 4(3)(-20)}}{2(3)}$$

$$= \frac{-7 \pm \sqrt{49 + 240}}{6} = \frac{-7 \pm \sqrt{289}}{6}$$

$$= \frac{-7 \pm 17}{6} = \begin{cases} \frac{10}{6} = \frac{5}{3} \\ \frac{-24}{6} = -4 \end{cases}$$

So the x-intercepts are $(-4, 0)$ and $\left(\frac{5}{3}, 0\right)$.




axis of symmetry?

$$\frac{1}{2}\left[-4 + \frac{5}{3}\right] = \frac{1}{2}\left[\frac{-12}{3} + \frac{5}{3}\right]$$

$$= \frac{1}{2}\left(\frac{-7}{3}\right) = \frac{-7}{6}$$

$$x = -\frac{7}{6}$$