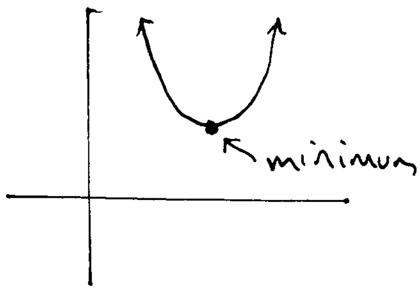
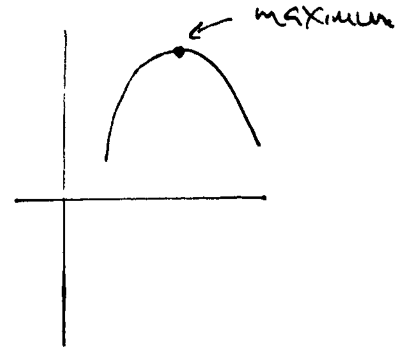
Math 96 Tue 10 Nov 2015

11.8 Max and Min problems

Idea: The graph of a quadratic function is a parabola
Finding a max or min is the same as finding the vertex.



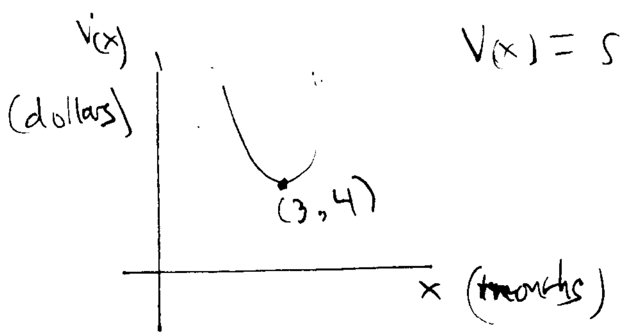


8) The value a share of stock is

$$V(x) = x^2 - 6x + 13$$

where x is the number of months after Jan. 2009.
what is the lowest value $V(x)$ will reach,
and when did that occur.

x = time (months)
$V(x)$ = stock price (dollars)

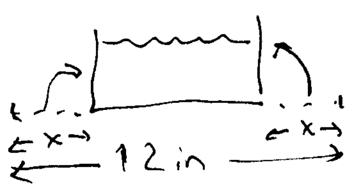


$$-\frac{b}{2a} = h = -\frac{(-6)}{2(1)} = 3 \text{ months}$$

$$V(3) = 3^2 - 6(3) + 13$$
$$= 9 - 18 + 13$$
$$= 4 \text{ dollars}$$

The minimum value was 4 dollars.
This happened at month 3 (so 3 months after Jan 2009 is Apr 2009)

ex: You design rain gutters out of 12 inch wide sheet metal by folding it so it has a rectangular cross-section, open top.

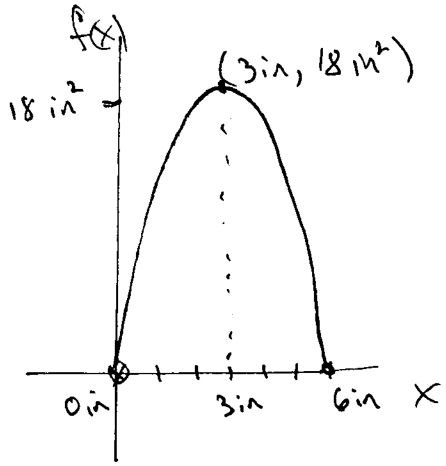


what should x be (in inches) to maximize the cross-sectional area.

Let $f(x) =$ area $=$ (base)(height)

where height $= x$
and base $= 12 - 2x$

so $f(x) = x(12-2x) = -2x^2 + 12x$ ← so $\begin{cases} a=-2 \\ b=12 \\ c=0 \end{cases}$

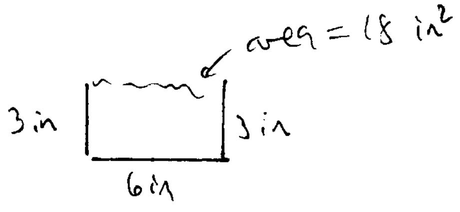


Graph? y-int of graph (0,0)

x-intercepts? Set $f(x)=0$ and solve for x:

$$0 = x(12-2x)$$

$x = 0$ inches or $12 - 2x = 0$
$12 = 2x$
$x = 6$ inches



$h = -\frac{b}{2a} = -\frac{12}{2(-2)} = 3$ inches

$k = 3(12 - 2\cdot 3) = 3 \cdot 6 = 18 \text{ in}^2$

what are the dimensions of the perfectly designed rain gutter?

$x = ht = 3$ inches , $12 - 2\cdot 3 = 6$ inches $=$ base

11.9 Polynomial and Rational Inequalities

For what x is x^2-6x+5 negative?

ex: $x^2-6x+5<0$

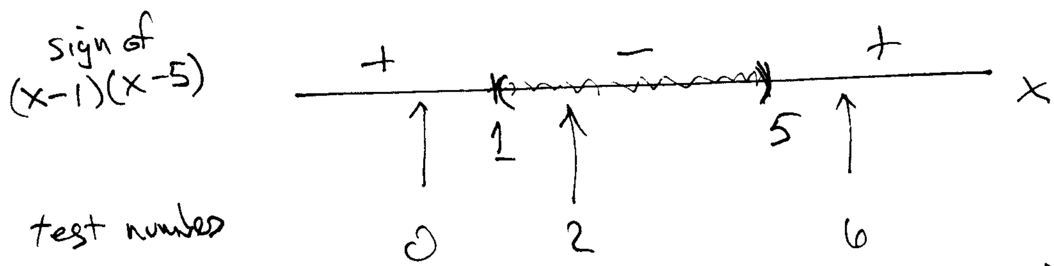
Step 1: Solve $x^2-6x+5=0$

$$(x-1)(x-5)=0$$

$$x-1=0 \quad \text{or} \quad x-5=0$$

$$x=1 \quad \text{or} \quad x=5$$

Step 2: Draw a "sign diagram" to keep track of the sign of x^2-6x+5 depending on x.



$(0-1)(0-5)>0$

$(6-1)(6-5)=5\cdot 1=5>0$

$(2-1)(2-5)=(1)(-3)=-3<0$

Answer: $(1,5)$ = open interval $=\{x \mid 1\le x\le 5\}$

ex: Solve $x^2+4x \le 2$

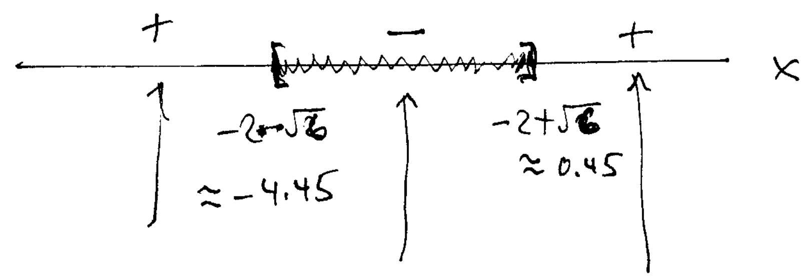
Equivalent: $x^2+4x-2 \le 0$

Step 1: Solve $x^2+4x-2=0$

$$x^2+4x+4 = 2+4$$
$$(x+2)^2 = 6$$
$$x+2 = \pm\sqrt{6}$$
$$x = -2 \pm \sqrt{6} \qquad x = -2 \pm 2.45$$

Step 2:

sign of x^2+4x-2

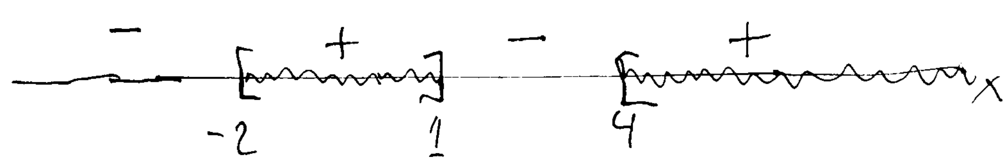


Test numbers $\quad -5 \quad -2 \quad 1$

Answer $= \left[-2-\sqrt{6}, -2+\sqrt{6}\right]$

25) Solve $(x-1)(x+2)(x-4) \ge 0$

Sign of $(x-1)(x+2)(x-4)$



Solution: $[-2, 1] \cup [4, \infty)$

38) (Rational inequality)

$$\frac{x-1}{x-2} \le 1$$

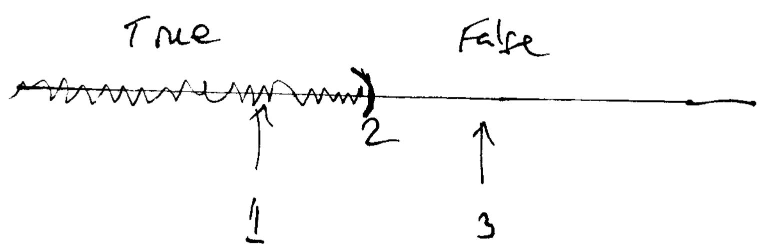
Is this undefined for any x? Yes if $x = 2$.

Solve $\frac{x-1}{x-2} = 1 \Rightarrow \frac{(x-1)\cancel{(x-2)}}{\cancel{x-2}} = 1 \cdot (x-2)$

$$\Rightarrow x - 1 = x - 2$$
$$-x \qquad -x$$
$$-1 = -2$$

No solution.

True/False ~~Sign~~ diagram



Test points:

Is $\frac{1-1}{1-2} \le 1$?

$0 \le 1$ is TRUE

Is $\frac{3-1}{3-2} \le 1$?

$2 \le 1$ is False

ANSWER: $(-\infty, 2)$

12.1 Composite Functions and Inverse Functions

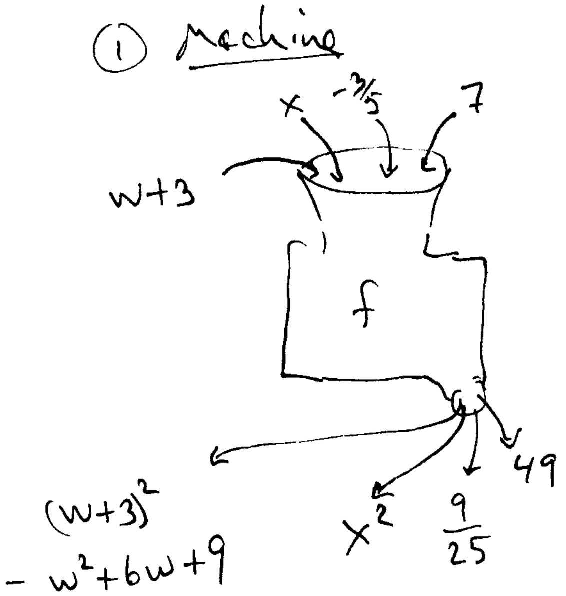
What is a function?

Five ways to think about functions

example: Imagine the "squaring function".

1. Machine
2. Equation
3. Table
4. Graph
5. Mapping

① Machine



② Equation

$f(x) = x^2$

$f(7) = 7^2 = 49$

$f(w+3) = w^2+6w+9$

or

$y = x^2$

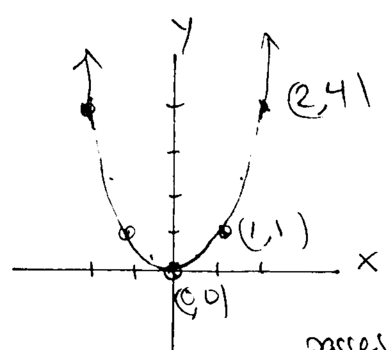
③ Table

x	x^2
-2	4
-1	1
0	0
1/2	1/4
1	1
2	4
3	9

Domain of f

Range of f

④ Graph



passes the vertical line test

5 <u>Mapping</u>