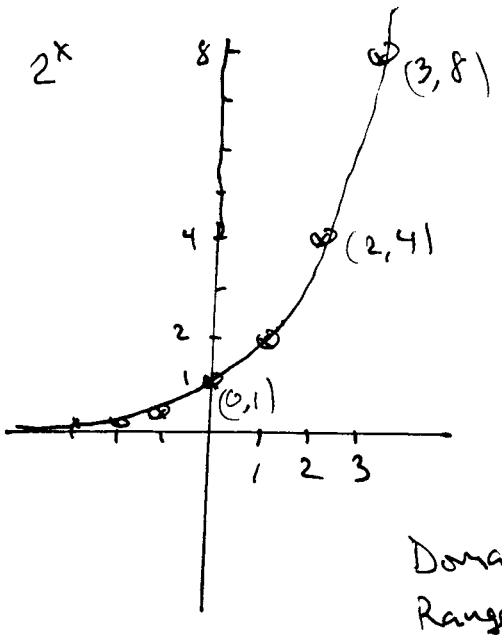
12.2 Exponential functions

ex. $f(x) = 2^x$

x	2^x
-3	1/8
-2	1/4
-1	1/2
0	1
1	2
2	4
3	8
4	16



$y = 2^x$ is a one-to-one function.

Domain $= (-\infty, \infty)$

Range $= (0, \infty)$

Note: Since base $= 2 > 1$ this is an increasing function.

If b = base < 1, $y = b^x$ is a decreasing function.

ex: (Interest compounded annually)

$$\boxed{A = P(1+r)^t}$$

P = principal (present value) = initial value of bank balance

r = interest rate (per year)

t = time (years)

A = amount (dollars) after t years.

Suppose $P = \$1000$ and $r = 6\% = .06$.

what is A after 0 years, 1 year, 2 years, 10 years?

If $t = 0$, $A = 1000(1+.06)^0 = 1000 \cdot 1.06^0 = 1000$ dollars

If $t = 1$ yr, $A = 1000(1+.06)^1 = 1000 \cdot 1.06 = 1060$ "

If $t = 2$ yrs, $A = 1000(1+.06)^2 = 1000 \cdot 1.06^2 = 1000(1.1236)$
$= \$1,123.60$

If $t = 10$ yrs, $A = 1000(1.06)^{10} = \$1,790.85$

12.3 Log functions

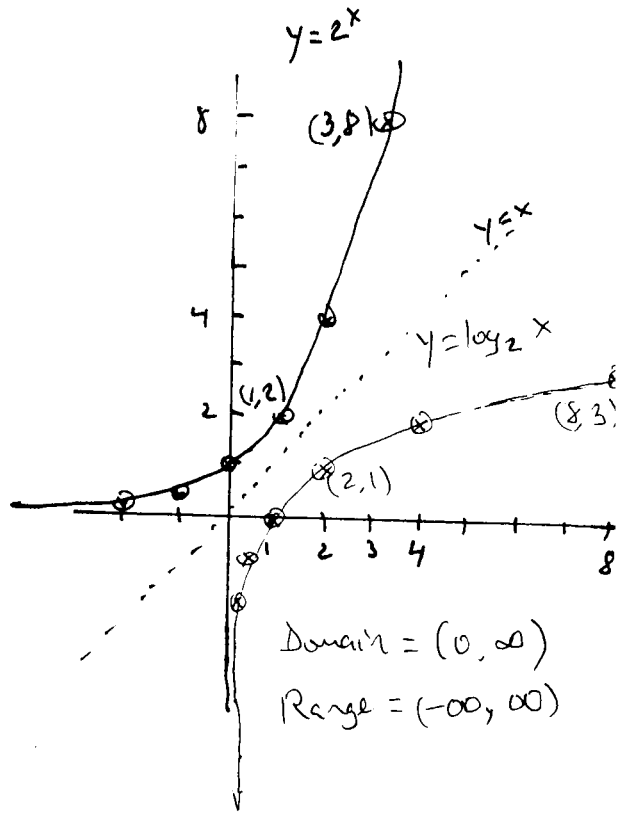
$\log_2(8) = 3$ means $2^3 = 8$

$\log_2(32) = 5$ means the same as $2^5 = 32$

$\boxed{\log_2(x) = y \text{ means } 2^y = x}$

y	$2^y = x$
-2	1/4
-1	1/2
0	1
1	2
2	4
3	8
4	16
5	32

x	$\log_2(x) = y$
1/4	-2
1/2	-1
1	0
2	1
4	2
8	3
16	4
32	5



Defn of logs:

$y = \log_b x$	means (or is equivalent to)	$b^y = x$

ex: Write the equivalent log equations:

$2^4 = 16$ means $\log_2 16 = 4$

(4 is the exponent, 16 is the other, 2 is the base)

$10^2 = 100$ is equivalent to $\log_{10} 100 = 2$

$3^4 = 81$ means $\log_3 81 = 4$

$10^{1/2} = \sqrt{10}$ means $\log_{10} \sqrt{10} = \frac{1}{2}$

$5^x = 125$ means $\log_5 125 = x$

ex: Write the equivalent exponent equations:

$\log_2 64 = 6$ means $2^6 = 64$

(64 is the other, 6 is the exp, 2 is the base)

$\log_7 7 = 1$ means $7^1 = 7$

$\log_5 625 = x$ mean $5^x = 625$

Hmm. That suggest that $x = 4$

So $\log_5 625 = 4$.

15) Evaluate $\log_7 49$.

Let $x = \log_7 49$.

This means $7^x = 49$

$7^x = 7^2$ so $\boxed{x=2}$

$\therefore \log_7 49 = \boxed{2}$

29) $\log_{10} 0.01$

Let $x = \log_{10} 0.01$

Equivalent $10^x = 0.01 = \frac{1}{100} = \frac{1}{10^2}$

$10^x = 10^{-2}$ so $x = -2$

$\therefore \log_{10} 0.01 = -2$

31) $x = \log_{16} 4$ Equiv: $16^x = 4$

$(4^2)^x = 4$

$4^{2x} = 4^1$

So $2x = 1$ so $x = \frac{1}{2}$

Answer: $\log_{16} 4 = \frac{1}{2}$ ← if we're to believe this,

check: $16^{1/2} = 4$ ← we should believe this.

Yes! $\sqrt{16} = 4$ ✓

ex. [log equation; unknown in any of the three slots]

i) $x = \log_3 81$ (exp) Solution: $3^x = 81 = 3^4$

So $x = 4$.

ii) $3 = \log_5 x$ (other) Solution: $5^3 = x$

So $x = 125$.

iii) $2 = \log_x 49$ Solution: $x^2 = 49$

$x = 7$ or ~~$x = -7$~~

bases are positive

Intro to calculator:

log means $\log_{10}$.

ex: $x = \log_{10} 10{,}000$ is equivalent to

$10^x = 10{,}000 = 10^4$

So $x = 4$.

ex. $x = \log_{10} \sqrt{10}$ means $10^x = \sqrt{10}$

$10^x = 10^{1/2}$

$x = \frac{1}{2} = .5$