Warmup

1. Write in log form: $5^{-3} = \frac{1}{125}$

$$\log_5\left(\frac{1}{125}\right) = -3$$

2. Find $\log_{16} 8$.

[Shortcut method] $\log_{16} 8 = \frac{\log_{10} 8}{\log_{10} 16} = \frac{\log_{10} 2^3}{\log_{10} 2^4}$

$$= \frac{3 \cdot \log_{10} 2}{4 \cdot \log_{10} 2} = \frac{3}{4}$$

[Text Book method] Let $x = \log_{16} 8$.

Equiv: $16^x = 8$

$$(2^4)^x = 2^3$$

$$2^{4x} = 2^3$$

$$4x = 3$$

$$x = 3/4$$

3. Find $\log_5 100$. $\log_5 100 = \frac{\log_{10} 100}{\log_{10} 5} = \frac{2}{0.69897}$

$$= 2.861353116$$

Loose ends in 12.5: Common log and Natural log

$\log 5$ means $\log_{10} 5$ ("common log")

≈ 0.69897

calculator

$\ln 10$ means $\log_e 10$

where $e = 2.718281828$

[Idea: e is such a famous number, there is a letter e permanently assigned to it. Just think $\pi = 3.141592\ldots$]

$$e = \left(1 + \frac{1}{\text{HUGE}}\right)^{\text{HUGE}}$$

ex: $\ln(e) = \log_e e^1 = 1$

$\ln e^{17} = \log_e(e^{17}) = 17$

12.6 Exponential and Log Equations

Exponential equations where we can get a common base
(no calculator needed)

ex: $8^x = 2^{x+1}$

$(2^3)^x = 2^{x+1}$

$2^{3x} = 2^{x+1}$

Now, because $f(x) = 2^x$ is a one-to-one function:

$3x = x + 1$

$2x = 1$

$\boxed{x = 1/2}$

ex: $25^x = 125^{x-4}$

$(5^2)^x = (5^3)^{x-4}$

$5^{2x} = 5^{3x-12}$

$2x = 3x - 12$

$0 = x - 12$

$\boxed{12 = x}$

ex: $5^{x^2} = 5^{-x+6}$

$x^2 = -x + 6$ → $x^2 + x - 6 = 0$

$(x+3)(x-2) = 0$

$x+3=0$ or $x-2=0$

$\boxed{x=-3 \text{ or } x=2}$

Exponential Equations where we cannot get a common base (calculator needed)

ex: $2^x = 7$

$$\ln 2^x = \ln 7$$

$$x \ln 2 = \ln 7$$

$$\frac{x \cancel{\ln 2}}{\cancel{\ln 2}} = \frac{\ln 7}{\ln 2}$$

$$x = \frac{\ln 7}{\ln 2}$$ "exact answer"

$$= \frac{1.94591}{0.69315}$$ "approximate answer to four decimal places"

$$= 2.8074$$

Remark 1 what if we'd used base 10 logs?

$$x = \frac{\log 7}{\log 2} = \frac{0.84510}{0.30103} = 2.8074$$

2 $2^x = 7$ is equivalent to

$$x = \log_2 7 = \frac{\ln 7}{\ln 2} = \frac{\log 7}{\log 2} = 2.8074$$

By change-of-base formula

12.6

18) Solve, to three decimal places,

$$5^{x+2} = 15$$

$$\ln 5^{x+2} = \ln 15$$

$$(x+2)\ln 5 = \ln 15$$

$$\frac{(x+2)\cancel{\ln 5}}{\cancel{\ln 5}} = \frac{\ln 15}{\ln 5}$$

$$x+2 = \frac{\ln 15}{\ln 5}$$

$$x = -2 + \frac{\ln 15}{\ln 5} = -2 + 1.6826$$

$$= \boxed{-0.317}$$

26)

$$29 = 3e^{2x}$$

$$\frac{29}{3} = \frac{\cancel{3}e^{2x}}{\cancel{3}}$$

$$\frac{29}{3} = e^{2x}$$

property (1) $\log_b b^y = y$ with $b=e$, $y=2x$.

Directly: equivalent log equation

$$\ln\left(\frac{29}{3}\right) = \ln e^{2x}$$

$$\ln\left(\frac{29}{3}\right) = 2x$$

$$\frac{1}{2}\ln\left(\frac{29}{3}\right) = \frac{1}{2}\cdot 2x$$

$$x = \frac{1}{2}\ln\left(\frac{29}{3}\right) \approx 1.134$$

exact — approximate

12.6 #27) $7 + 3e^{-x} = 13$

$-7 \qquad -7$

$3e^{-x} = 6$

$\frac{3e^{-x}}{3} = \frac{6}{3}$

$e^{-x} = 2$

$\ln e^{-x} = \ln 2$

$-x = \ln 2$

$x = -\ln 2 = -0.693$

<u>Log equations:</u> log on <u>one</u> side only.

<u>ex</u>: $\log_2 x = 3$

$x = 2^3 = 8$

<u>ex</u>: $\log_5 x = 1.7$

$x = 5^{1.7} \approx 15.426$

<u>ex</u>: $\log_3 (x-1) = 4 \rightarrow 3^{\log_3 (x-1)} = 3^4$

$x-1 = 3^4 = 81$

$\boxed{x = 82}$

because of property ②:

$a^{\log_a y} = y$

with $a=3$, $y=x-1$.

Remark: 1 what is the domain of

$$f(x) = \log_3 (x-1) \ ?$$

That is, what numbers are legal to substitute for x?

we must have $x-1>0$, so $x>1$.

2 we can have extraneous solutions when dealing with log equations.

12.6 48) $\log(x+9) + \log x = 1$

must have $x+9>0$ so $x>-9$

$\boxed{\text{Must have } x>0}$.

First step: Combine logs:

$$\log[(x+9)x] = 1$$

by property 3: product rule

$$\log(x^2+9x) = 1$$

$$x^2+9x = 10^1$$

$$x^2+9x = 10$$

$$x^2+9x-10=0$$

$$(x+10)(x-1)=0$$

$x+10=0$ or $x-1=0$

$\cancel{x=-10}$ $\qquad \boxed{x=1}$

extraneous

53) $\log_4(x+3) = 2 + \log_4(x-5)$

Must have $x+3>0$ or $x>-3$

Must have $x-5>0$ $\boxed{x>5}$

$\log_4(x+3) - \log_4(x-5) = 2$ Combine into a single log.

$\log_4\left(\frac{x+3}{x-5}\right) = 2$

$\frac{x+3}{x-5} = 4^2 = 16$

$\frac{\cancel{(x-5)}}{1} \cdot \frac{(x+3)}{\cancel{(x-5)}} = 16(x-5)$

$x+3 = 16x - 80$
$-x \quad\quad -x$

$3 = 15x - 80$

$83 = 15x$

$\boxed{\frac{83}{15} = x}$ and $\frac{83}{15} > 5 = \frac{75}{15}$

Logs on both sides (all terms)

57) $\log_5(x+4) + \log_5(x-4) = \log_5 20$

must have $x > 4$.

$$\log_5[(x+4)(x-4)] = \log_5 20$$

$$\log_5(x^2-16) = \log_5 20$$

$$x^2 - 16 = 20$$

$$+16 \quad +16$$

$$x^2 = 36$$

$$x = \pm\sqrt{36}$$

$\boxed{x = 6}$ or $\cancel{x = -6}$