Warm-up

1. $$4^{x+3} = 8^x$$

$$(2^2)^{x+3} = (2^3)^x$$

$$2^{2x+6} = 2^{3x}$$

$$2x+6 = 3x$$
$$-2x \qquad -2x$$

$$\boxed{6 = x}$$

2. $$4^{2x-5} = 19$$

p844 #58)

$$\ln 4^{2x-5} = \ln 19$$ ← OR use common log

$$(2x-5)\ln 4 = \ln 19$$ Divide both sides by $\ln 4$:

$$2x-5 = \frac{\ln 19}{\ln 4}$$

$$2x = 5 + \frac{\ln 19}{\ln 4}$$ ← OR use common log

$$x = \frac{1}{2}\cdot 2x = \frac{1}{2}\left(5 + \frac{\ln 19}{\ln 4}\right) = \frac{1}{2}\left(5 + \frac{2.9444}{1.3863}\right)$$

$$= \frac{1}{2}\left(5 + 2.1240\right)$$

$$= \frac{1}{2}(7.1240) = \boxed{3.5620}$$

3

#60)

$$e^{-0.1t} = 0.03$$

$$\ln e^{-0.1t} = \ln 0.03$$

$$-0.1t = \ln 0.03$$

$$t = \frac{\ln 0.03}{-0.1} = \frac{-3.5066}{-0.1} = \boxed{35.0656}$$

4

#63)

$$\log_4 x - \log_4 (x-15) = 2$$ Must have $x > 15$.

$$\log_4 \left(\frac{x}{x-15}\right) = 2$$

$$\frac{x}{x-15} = 4^2 = 16$$

$$\frac{x(x-15)}{(x-15)} = 16(x-15)$$

$$x = 16x - 240$$

$$-16x \quad -16x$$

$$-15x = -240$$

$$\frac{15x}{15} = \frac{240}{15}$$

$$\boxed{x = 16}$$ and $16 > 15$.

5

#64) $\log_3(x-4) = 2 - \log_3(x+4)$

Must have $x > 4$

$+\log_3(x+4) \qquad +\log_3(x+4)$

$\log_3(x-4) + \log_3(x+4) = 2$

$\log_3[(x-4)(x+4)] = 2$

$\log_3(x^2-16) = 2$

$x^2 - 16 = 3^2$

$x^2 - 16 = 9$

$+16 \quad +16$

$x^2 = 25$

$x = \pm\sqrt{25} = \pm 5$

$\boxed{x = 5}$ OR $\cancel{x = -5}$

12.7 Applications

continuous compounding of interest

margin 6) (p 831)

$$\boxed{P(t) = P_0 e^{kt}}$$

compare with (p 814)

$$\boxed{A = P\left(1+\frac{r}{n}\right)^{nt}}$$

$P(0) = P_0$ = principal [same as P]

k = annual interest rate [same as r]

t = time (years)

$P(t)$ = amount after t years [same as A]

P = principal (or present value)

r = annual interest rate

n = number of times per year of compounding (if monthly, $n=12$; if daily, $n=365$)

t = time (years)

A = amount (in dollars) after t years.

example: In ~~an~~ account with continuous compounding,

$P(0) = P_0 = \$20,000$ and

$P(5) = \$22,103.42$.

a) what is the growth function?

b) what is the doubling time? That is, for what t is $P(t) = \$40,000$?

solution: a) we need to learn what P_0 and k are equal to.

Given: $P_0 = \$20,000$

we will find k by indirect means.

$$P(t) = 20{,}000\, e^{kt}$$

$$22{,}103.42 = P(5) = 20{,}000\, e^{k \cdot 5}$$

$$\frac{22{,}103.42}{20{,}000} = \frac{\cancel{20{,}000}\, e^{5k}}{\cancel{20{,}000}}$$

$$1.105171 = e^{5k}$$

$$\ln 1.105171 = \ln e^{5k}$$

$$0.1 = 5k$$

$$\frac{0.1}{5} = \frac{5k}{5}$$

$$2\% = 0.02 = k$$

$$\boxed{P(t) = 20{,}000\, e^{0.02t}}$$

solution b) $$\frac{40{,}000}{20{,}000} = \frac{20{,}000\, e^{0.02t}}{20{,}000}$$

$$2 = e^{0.02t}$$

$$\ln 2 = \ln e^{0.02t}$$

$$\ln 2 = 0.02t$$

$$\frac{\ln 2}{0.02} = \frac{0.02t}{0.02}$$

$$\frac{0.693147}{0.02} = t$$

$\boxed{t = 34.66 \text{ years}}$

is the doubling time