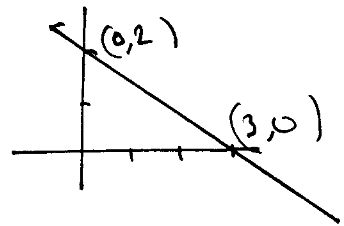
# Chapter 13 – Conic Sections

Theme: Graph $\rightleftarrows$ Standard Equation $\leftarrow$ Nonstandard Equation

(Analytic Geometry)

example. (Line)



$\xrightarrow{\text{equation?}}$ $\boxed{y = mx + b}$

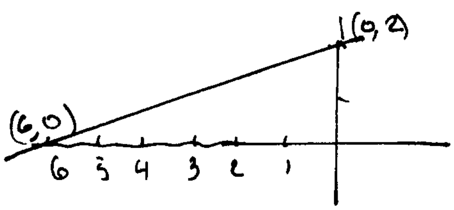
where $m$ = slope $= \frac{-2}{3}$

$b$ = y-intercept $= 2$

$$y = -\frac{2}{3}x + 2$$

ex: Graph: $-x + 3y = 6$ $\xrightarrow{\text{std eqn?}}$ $3y = x + 6$

$$y = \frac{1}{3}x + 2$$



$\xrightarrow{\text{graph?}}$ slope $= 1/3$

y-intercept $= 2$

## 13.1 Circles

standard equation of a circle

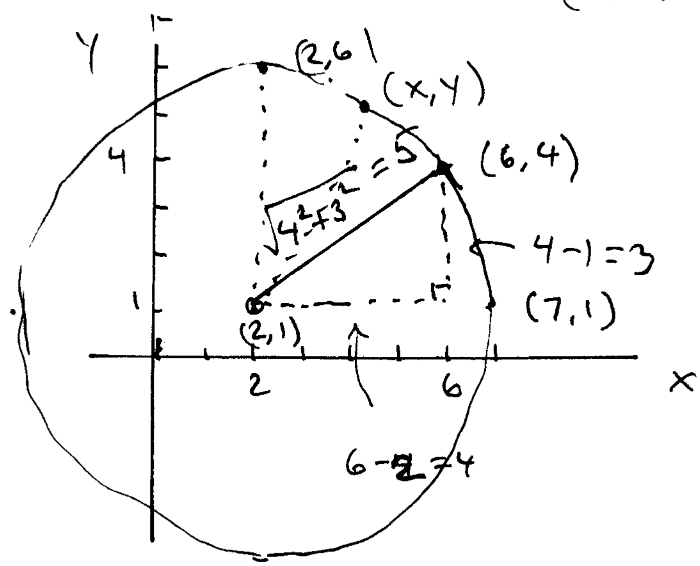
$$\boxed{(x-h)^2 + (y-k)^2 = r^2}$$

center $= (h, k)$

radius $= r$

ex (ignore the equation for now)

what are the coordinates $(x, y)$ of points which lie 5 units from $(2, 1)$?




Distance formula

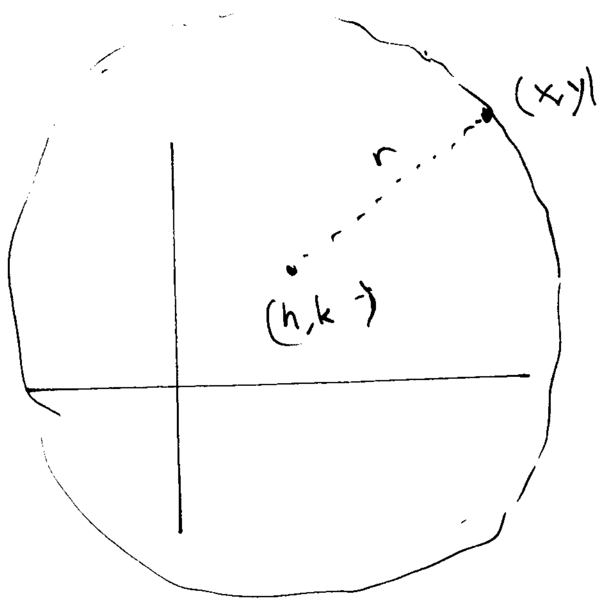
$$\text{distance} = \sqrt{(x-2)^2 + (y-1)^2} = 5$$

OR

$$\boxed{(x-2)^2 + (y-1)^2 = 25}$$

$5^2$

The equation of a circle with center $= (2,1)$ and radius $= 5$.



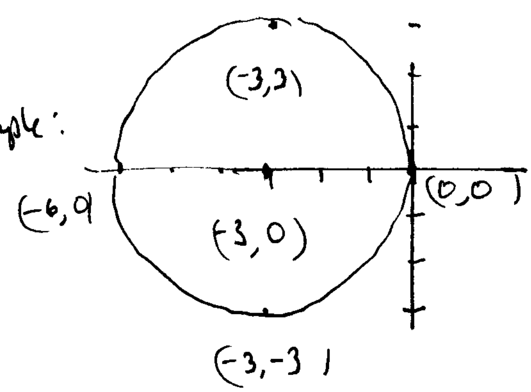

what can we say about $(x,y)$ if $(x,y)$ lies $r$ units from $(h,k)$?

Answer: Std. eqn. of a circle

$$\boxed{(x-h)^2 + (y-k)^2 = r^2}$$

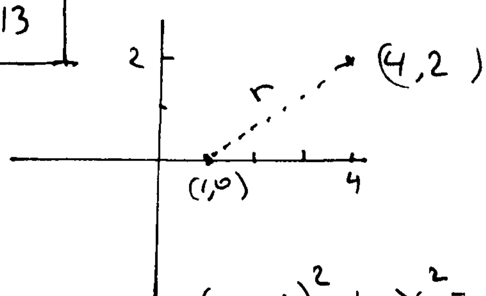
| Equation | Center | Radius |
| --- | --- | --- |
| $(x-3)^2+(y-5)^2=49 \rightarrow$ | $(3,5)$ | $7$ |
| $(x+2)^2+y^2=25 \rightarrow$ | $(-2,0)$ | $5$ |
| $(x-\frac{3}{2})^2+(y+\frac{1}{2})^2=8 \rightarrow$ | $(\frac{3}{2}, -\frac{1}{2})$ | $\sqrt{8}=2\sqrt{2}$ (because $r^2=8$) |
| $(x-2)^2+(y-4)^2=36 \leftarrow$ | $(2,4)$ | $6$ |
| $(x+3)^2+y^2=9$ $\leftarrow$ | $(-3,0)$ | $3$ |

Graph of last example:




example: A circle has center at $(1,0)$ and passes through the point $(4,2)$. Find its equation.

Answer:
$\boxed{(x-1)^2+y^2=13}$




Need: $h, k, r$.

$(h,k)=(1,0)$

Find $r$? Use the pythagorean theorem or distance formula

OR

$(x-1)^2+y^2=r^2$ Use that $(4,2)$ will satisfy the equation:

$r^2=(4-1)^2+2^2=3^2+2^2=9+4=13$ so $r=\sqrt{13}$

ex [Non-standard equation of a circle]

55) Given $x^2+y^2+8x-6y-15=0$

show that this <u>is</u> the equation of a circle and find its center and radius. [Method: Complete the square.]

$$x^2+8x \quad + y^2-6y \quad = 15$$

$$x^2+8x+16 \quad + y^2-6y+9 \quad = 15+16+9$$

standard equation →

$$(x+4)^2 + (y-3)^2 = 40$$

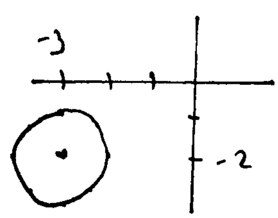
center $= (-4, 3)$ radius $= \sqrt{40} = 2\sqrt{10}$
$\approx 6.32$

58) Same for $x^2+y^2+6x+4y+12=0$

$$x^2+6x+9 + y^2+4y+4 = -12+9+4$$

$$(x+3)^2 + (y+2)^2 = 1$$

center $= (-3,-2)$ radius $= 1$



## Parabolas

$$\boxed{y = a(x-h)^2+k}$$

OR

$$\boxed{x = a(y-k)^2+h}$$

$(h,k)$ = vertex

If $a>0$, open up.
If $a<0$, open down.

$(h,k)$ = vertex

If $a>0$ open to right (positive x-direction)

If $a<0$ open to left

ex: Find the vertex of $y=-(x-2)^2+1$ and sketch the graph

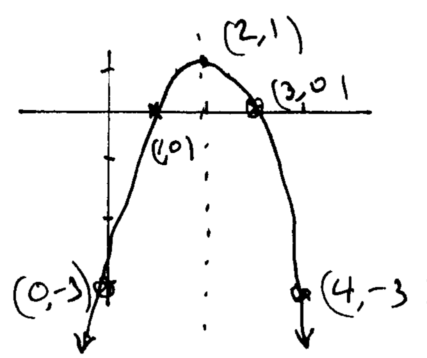
vertex? For what $x$ is $x-2=0$? Answer: $x=2$.

when $x=2$, what is $y$? $y=-(2-2)^2+1 = 1$

vertex $= (2,1)$

| x | y |
|---|---|
| 0 | -3 |
| 1 | 0 |
| 2 | 1 |
| 3 | 0 |
| 4 | -3 |

$h=2$
$k=1$
$a=-1$

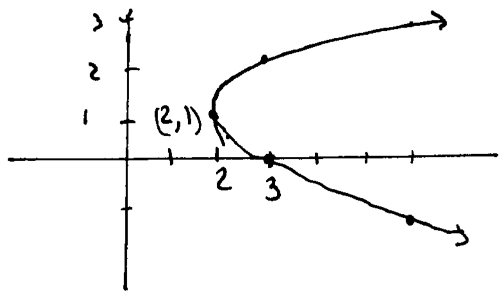



ex: $x=(y-1)^2+2$

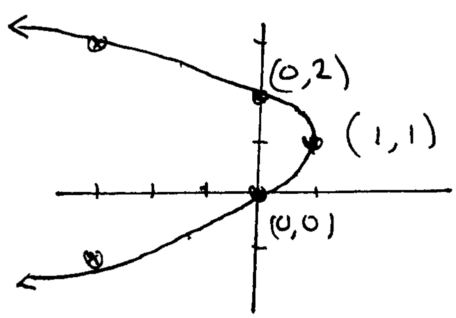
$a=1$ open to right
$h=$
$k=$

vertex? what $y$ makes $(y-1)=0$?
Answer: $y=1$
when $y=1$ what is $x$?
$x=(1-1)^2+2=2$
vertex: $(x,y)=(2,1)$

| | X | Y |
|---|---|---|
| | 6 | -1 |
| | 3 | 0 |
| vertex → | 2 | 1 |
| | 3 | 2 |
| | 6 | 3 |




example: Find an equation with this (parabolic) graph.




a will be negative

$x = a(y-k)^2 + h$

Need: $h, k, a$

$(h,k) = (1,1)$

$x = a(y-1)^2 + 1$

Use that $(0,0)$ satisfies the equation:

This equation is true → $0 = a(0-1)^2 + 1$

$0 = a + 1 \Rightarrow a = -1$

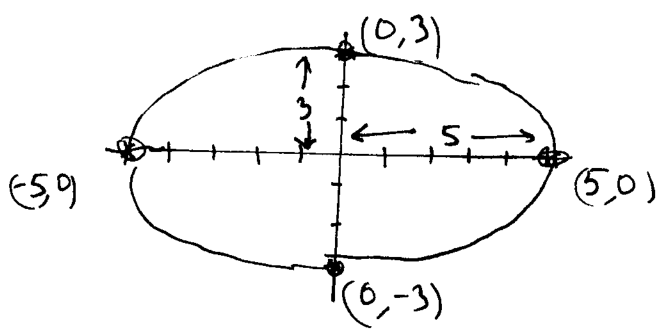
$$\boxed{x = -(y-1)^2 + 1}$$

Brief intro to Ellipses (§13.2)

$$\boxed{\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1} \quad \text{or} \quad \boxed{\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1}$$

where $a$ = length of semimajor axis
$b$ = length of semiminor axis

ex $$\frac{x^2}{25} + \frac{y^2}{9} = 1$$




$a = 5$
$b = 3$