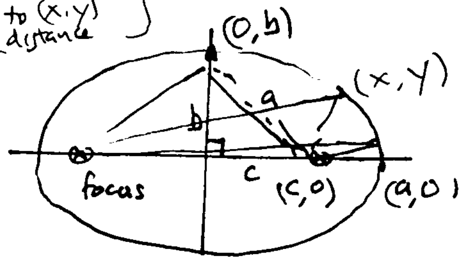
13.2 Ellipses (cont'd)

$\left\{\begin{matrix}\text{1st focus} \\ \text{to } (x,y) \\ \text{distance}\end{matrix}\right\} + \left\{\begin{matrix}\text{2nd focus} \\ \text{to } (x,y) \\ \text{distance}\end{matrix}\right\} = 2a$



$2a$ = length of string

a = length of semimajor axis

b = " semiminor "

c = center-to-focus distance

Note: $a^2 = b^2 + c^2$

Ellipse $\boxed{\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1}$

center at origin $a > b$

longer in the x-direction

x-intercepts $= (\pm a, 0)$

y-intercepts $= (0, \pm b)$

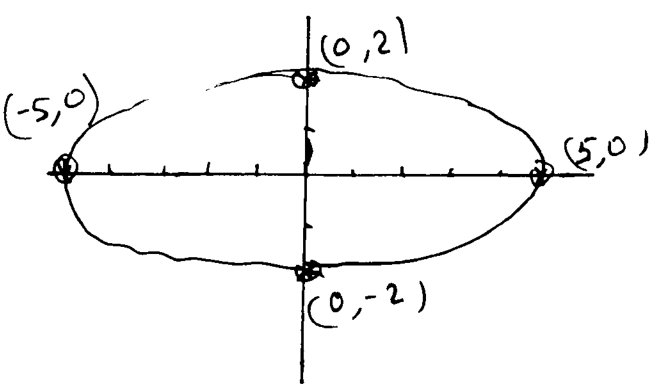
ex: [equ → graph] Graph $\dfrac{x^2}{25} + \dfrac{y^2}{4} = 1$.

x-intercepts? Set $y=0$, solve for x:

$$\frac{x^2}{25} + \cancel{\frac{0^2}{4}}^{\,0} = 1 \Rightarrow \frac{x^2}{25} = 1$$

$$x^2 = 25$$

$$x = \pm\sqrt{25} = \pm 5$$

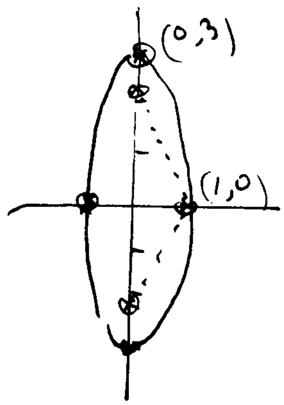


y-intercepts? Set $x=0$, solve for y:

$$\frac{0^2}{25} + \frac{y^2}{4} = 1 \Rightarrow \frac{y^2}{4} = 1 \Rightarrow y^2 = 4$$

$$y = \pm 2$$

ex: Graph $x^2 + \frac{y^2}{9} = 1$ Identify a and b.



x-intercepts? $(\pm 1, 0)$

y-intercepts? $(0, \pm 3)$

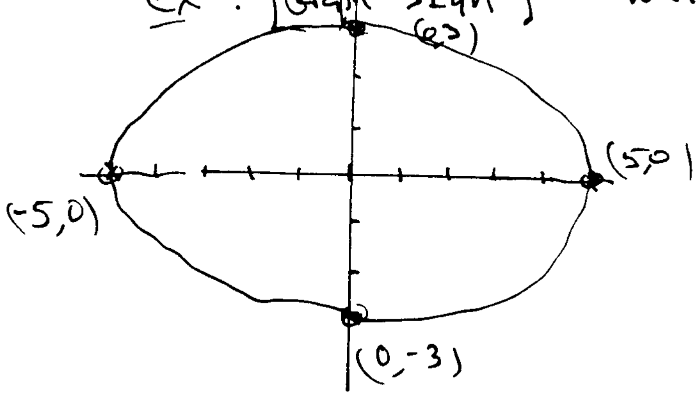
$a = 3$ $b = 1$

NOTE: This has the form

$$\boxed{\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1}$$

Ellipse
center at (0,0)
longer in y-direction

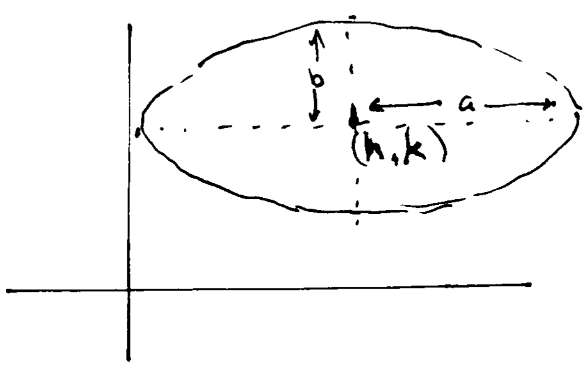
ex: [Graph → Eqn] what is the equation of this ellipse?



$a = 5$ $b = 3$

$a^2 = 25$ $b^2 = 9$

$$\boxed{\frac{x^2}{25} + \frac{y^2}{9} = 1}$$



$$\boxed{\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1}$$

ellipse

center = (h, k)

semimajor = a

semiminor = b

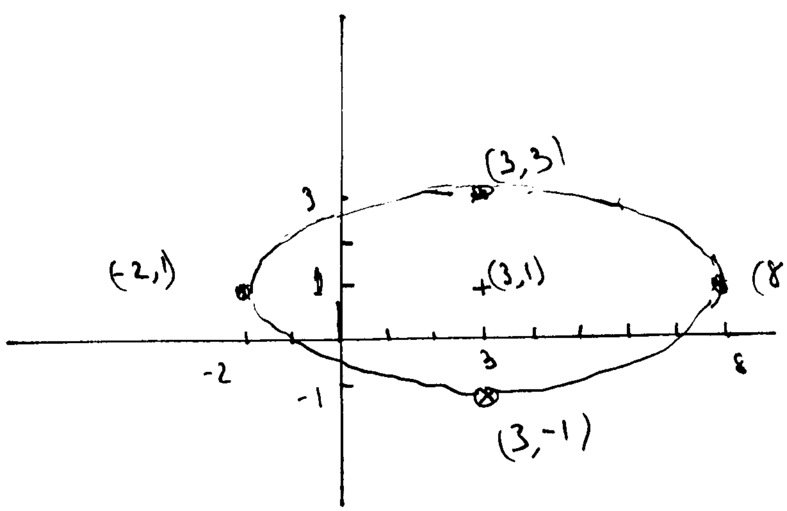
ex [Eqn → graph] Graph

$$\frac{(x-3)^2}{25} + \frac{(y-1)^2}{4} = 1$$

$a = 5$
$b = 2$
$h = 3$
$k = 1$

center $= (h,k) = (3,1)$

$5 = a =$ semimajor axis
$2 = b =$ semiminor "



vertices at
$(8,1)$ and $(-2,1)$

Summary

$$\boxed{\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1}$$

longer in x-direction

OR

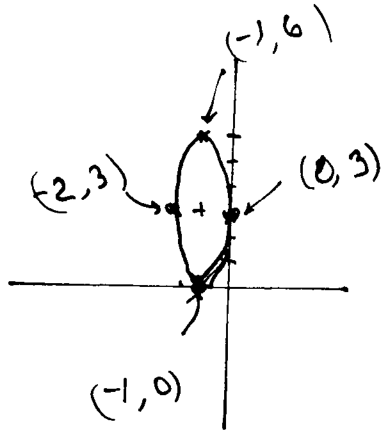
$$\boxed{\frac{(x-h)^2}{b^2} + \frac{(y-k)^2}{a^2} = 1}$$

longer in y-direction.

Center $= (h,k)$
$a =$ semimajor
$b =$ semiminor

ex: [Graph → Equ]

Find the equation.



$$(h,k) = (-1, 3)$$
$$a = 3$$
$$b = 1$$

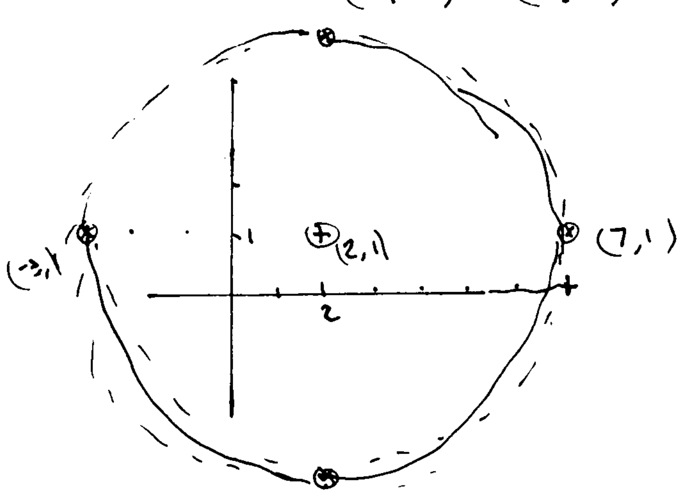
$$\frac{(x+1)^2}{1} + \frac{(y-3)^2}{9} = 1$$

check: $(0,3) = (x,y)$ satisfies
$$(x+1)^2 + \frac{(y-3)^2}{9} = 1$$

for $$(0+1)^2 + \frac{(3-3)^2}{9} = 1 + 0 = 1 \checkmark$$

So $(0, 3)$ is indeed a point on the ellipse.

Remark: what if
$(h,k) = (2,1)$ and $a = b = 5$?



$$\frac{(x-2)^2}{25} + \frac{(y-1)^2}{25} = 1$$

multiply both sides by 25:

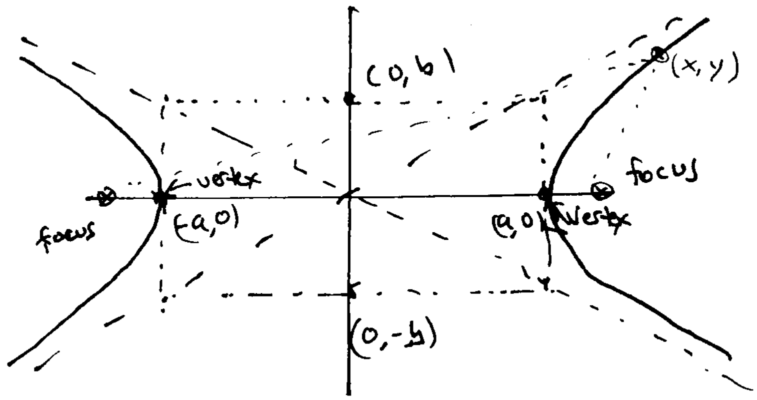
$$(x-2)^2 + (y-1)^2 = 25$$

that is, a circle
with center $= (2,1)$
radius $= 5$.

13.3 Hyperbolas

<u>Difference</u> of distances is a fixed number $= 2a$

$$\boxed{\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1}$$



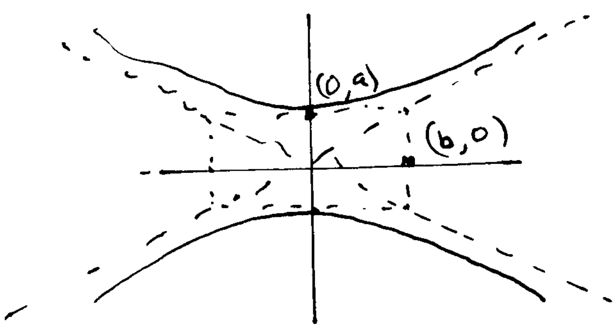
Hyperbola,
center at $(0,0)$
x-intercepts (vertices)
at $(\pm a, 0)$

~~transvers~~
transverse axis = line segment connecting the vertices : (has length $2a$)
conjugate axis = " " " $(0,b)$ with $(0,-b)$: (has length $2b$)

asymptotes : the diagonal lines

<u>OR</u>

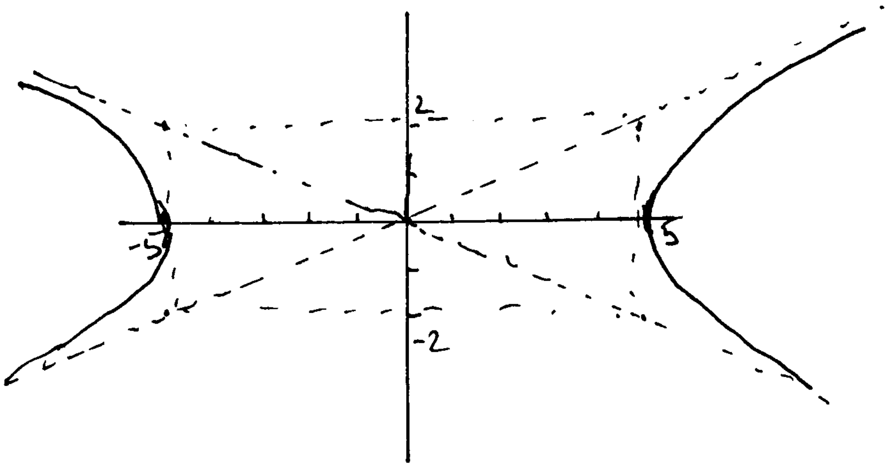
$$\boxed{\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1}$$



Hyperbola, center at $(0,0)$
y-intercepts at $(0, \pm a)$

<u>NOTE</u>: a^2 is under the <u>positive</u> term
So <u>a</u> may be smaller or larger or equal to <u>b</u>.

ex: [Equ → graph] Graph $\frac{x^2}{25} - \frac{y^2}{4} = 1$



x-intercepts? Set $y=0$, solve for x:

$\frac{x^2}{25} = 1$ so $x = \pm 5$

y-intercepts? Set $x=0$

$-\frac{y^2}{4} = 1 \Rightarrow y^2 = -4$

No real solutions

No y-intercepts

vertices: $(\pm 5, 0)$