Math 96 — Tuesday 10 May 2016

Supplemental notes

13.3 (A little more on) Hyperbolas

16) Graph, label all vertices, sketch all asymptotes.

$$4y^2 - 9x^2 = 36$$

Hyperbola, but not in standard equation.

$$\frac{4y^2}{36} - \frac{9x^2}{36} = \frac{36}{36}$$

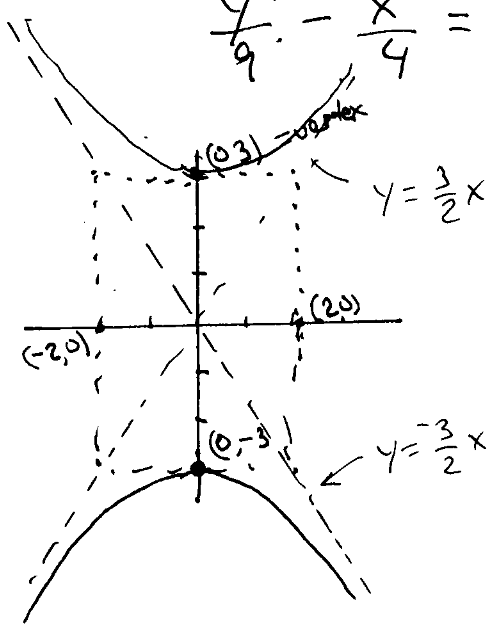
$$\frac{y^2}{9} - \frac{x^2}{4} = 1$$

← Has y-intercept, solve $\frac{y^2}{9} = 1$

$y^2 = 9$

$y = \pm 3$

And no x-intercept because $-\frac{x^2}{4} = 1$ has no real solution.



The equation

$$\frac{y^2}{9} - \frac{x^2}{4} = 1$$

has the form $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

where $\begin{cases} a = 3 \\ b = 2 \end{cases}$

∴ Half the length of the transverse axis = 3

and Half the length of the conjugate axis = 2

Classify the equation.
What type of curve is this?

27) $9x^2+4y^2-36=0$ ellipse

29) $4x^2-9y^2-72=0$ hyperbola

31) $y^2=20-x^2$

Hint: $x^2+y^2=20$ is equivalent circle

28) $x+3y=2y^2-1$ parabola (sidewise)

ex: $2x+3y-24=0$ line

32) $2y+13+x^2=8x-y^2$ circle

Equivalent: $x^2+y^2-8x+2y+13=0$ Center and ~~radius~~? radius?

$x^2-8x+16+y^2+2y+1=-13+16+1$

$(x-4)^2+(y+1)^2=4$ so center = $(4,-1)$
radius = 2

13.4 Nonlinear systems of equations

ex: [Example 1 in textbook] Solve the system

$$\begin{cases} x^2+y^2=25 \\ 3x-4y=0 \end{cases}$$

$x^2+y^2=25$ ← circle, { radius = 5, center = (0,0) }

$3x-4y=0$ ← line $y=\frac{3}{4}x$ slope $=\frac{3}{4}$ passes through (0,0)

Solve by substitution: Sub $y=\frac{3}{4}x$ into the circle equation

$$x^2+\left(\frac{3}{4}x\right)^2=25$$

← Single equation, one variable (we eliminated y)

$$x^2+\frac{9}{16}x^2=25$$

$$\frac{16x^2}{16}+\frac{9x^2}{16}=25$$

$$\frac{25x^2}{16}=25$$

Divide both sides by 25

$$\frac{x^2}{16}=1$$

$$x^2=16 \Rightarrow x=\pm\sqrt{16}=\pm 4$$

Back substitute: Case 1: $x=4$

$$y=\frac{3}{4}(4)=3$$

$$(x,y)=(4,3)$$

Case 2: $x=-4$

$$y=\frac{3}{4}(-4)=-3$$

$$(x,y)=(-4,-3)$$