1.2 Gaussian Elimination and Gauss-Jordan Elimination

ex.:

$$
\begin{aligned}
-y + 2z &= 4 \\
x \qquad + 3z &= 10 \\
2x + y + 4z &= 16
\end{aligned}
$$

Augmented matrix:

$$
\left[\begin{array}{ccc|c}
0 & -1 & 2 & 4 \\
1 & 0 & 3 & 10 \\
2 & 1 & 4 & 16
\end{array}\right]
$$

Elementary Row Operations

1. Swap any two rows.
2. Multiply a row by a (nonzero) constant
3. Add a multiple of (a copy of) one row to a second row, thereby modifying the second row.

Goal of Gaussian elimination: "row echelon form"

Goal of Gauss-Jordan elimination: "reduced row echelon form"

$$R_1 \leftrightarrow R_2 \quad \begin{bmatrix} 1 & 0 & 3 & 10 \\ 0 & -1 & 2 & 4 \\ 2 & 1 & 4 & 16 \end{bmatrix}$$

$$R_3 + (-2)R_1 \to R_3 \quad \begin{bmatrix} 1 & 0 & 3 & 10 \\ 0 & -1 & 2 & 4 \\ 0 & 1 & -2 & -4 \end{bmatrix}$$

$$(-1)R_2 \to R_2 \quad \begin{bmatrix} 1 & 0 & 3 & 10 \\ 0 & 1 & -2 & -4 \\ 0 & 1 & -2 & -4 \end{bmatrix}$$

$$R_3 + (-1)R_2 \to R_3 \quad \begin{bmatrix} 1 & 0 & 3 & 10 \\ 0 & 1 & -2 & -4 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\begin{aligned} x \qquad + 3z &= 10 \\ y - 2z &= -4 \end{aligned} \qquad \text{Let } z = t$$

Then
$$\begin{aligned} x &= -3t + 10 \\ y &= 2t - 4 \\ z &= t \end{aligned}$$

where t is any real number.

is the solution set.

Q: Can a system have 3 variables and 2 eqns and be consistent?

Yes: $\begin{cases} x + 3z = 10 \\ y - 2z = -4 \end{cases}$

rref = $\left[\begin{array}{ccc|c} 1 & 0 & 3 & 10 \\ 0 & 1 & -2 & -4 \end{array}\right]$

Q: Can a system have 3 variables, 2 eqns and be inconsistent?

Yes: $\begin{cases} x + y + z = 5 \\ x + y + z = 7 \end{cases}$

rref = $\left[\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right]$

Q: Can a system have 2 variables, 3 eqns and be consistent?

Yes:
$$x + y = 3$$
$$x + 10y = 21$$
$$2x + y = 4$$

has solutions $(x, y) = (1, 2)$

rref = $\left[\begin{array}{cc|c} 1 & 0 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{array}\right]$

Q: Can a system have 2 variables, 3 equations and be inconsistent?

YES:
$$x + y = 6$$
$$-x + y = 2$$
$$-x + 2y = 0$$

rref = $\left[\begin{array}{cc|c} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{array}\right]$