Math 254 Thu 21 June 2018

2.1 Matrix operations

$$\begin{bmatrix} 2 & -\frac{1}{2} & 4 & 7 & 0 \\ 5 & 0 & \pi & 0 & 3 \\ 4 & 8 & \sqrt{2} & 0 & 2 \end{bmatrix} \leftarrow 3\times 5 \text{ matrix}$$

3 rows
5 columns

$$\begin{bmatrix} 5 \\ 4 \\ -1 \\ 0 \end{bmatrix} \leftarrow 4\times 1 \text{ matrix} \qquad \begin{bmatrix} 3 & 5 & 8 & 4 \\ 2 & 6 & -1 & 0 \end{bmatrix} \leftarrow 2\times 4$$

<u>Equality</u> of matrices: Two matrices are equal if they are of the same size, and corresponding entries are equal.

$$\underset{2\times 1}{\begin{bmatrix} 2x+3y \\ x-4y \end{bmatrix}} = \begin{bmatrix} 5 \\ 7 \end{bmatrix} \leftarrow 2\times 1$$

says $2x+3y=5$ <u>and</u> $x-4y=7$

<u>Addition</u>: One can add matrices of the same size.

$$\begin{bmatrix} 3 & 5 \\ 0 & 4 \\ 1 & 2 \end{bmatrix} + \begin{bmatrix} 2 & -5 \\ 1 & -4 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 0 \\ 1 & 0 \\ 0 & 4 \end{bmatrix}$$

Scalar multiplication

$$10 \begin{bmatrix} 3 & 5 \\ 0 & 4 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 30 & 50 \\ 0 & 40 \\ 10 & 20 \end{bmatrix}$$

$$-1 \begin{bmatrix} 4 & 2 \\ 8 & 3 \end{bmatrix} = \begin{bmatrix} -4 & -2 \\ -8 & -3 \end{bmatrix}$$

Matrix multiplication

Defn: If $A = [a_{ij}]$ is an $m \times n$ matrix

and $B = [b_{ij}]$ is an $n \times p$ matrix

then the matrix product $C = AB$ is an $m \times p$

matrix $C = [c_{ij}]$ where

$$c_{ij} = \sum_{k=1}^{n} a_{ik} b_{kj}$$

$$= a_{i1} b_{1j} + a_{i2} b_{2j} + \cdots + a_{in} b_{nj} .$$

Warning: Matrix multiplication is NOT commutative in general!

Let $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ $B = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$

$$AB = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 3 \end{bmatrix}$$

$$BA = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 3 & 4 \\ 0 & 0 \end{bmatrix}$$

$AB \neq BA$

ex [Application] Suppose grades are calculated as

HW	10%	= 0.10
Q	10%	= 0.10
(two) Tests	24% each	= 0.24
(3rd) Test	8%	= 0.08
Final	24%	= 0.24

$$\begin{array}{l} 1^{st} \text{ student} \rightarrow \\ 2^{nd} \rightarrow \\ 3^{rd} \rightarrow \end{array} \begin{bmatrix} 100 & 100 & 100 & 100 & 100 & 100 \\ 50 & 50 & 50 & 50 & 50 & 50 \\ 100 & 100 & 100 & 100 & 40 & 100 \end{bmatrix} \begin{bmatrix} 0.10 \\ 0.10 \\ 0.24 \\ 0.24 \\ 0.08 \\ 0.24 \end{bmatrix} = \begin{bmatrix} 100 \\ 50 \\ 95.2 \end{bmatrix}$$

scratch work

$100 \times .10 = 10$
$100 \times .10 = 10$
$100 \times .24 = 24$
$100 \times .24 = 24$
$40 \times .08 = 3.2$
$100 \times .24 = 24$

95.2

ex. Represent the system using matrix algebra:

$$3x + 5y = 6$$
$$4x + 6y = -3$$

Let $A = \begin{bmatrix} 3 & 5 \\ 4 & 6 \end{bmatrix}$ let $\mathbb{x} = \begin{bmatrix} x \\ y \end{bmatrix}$ and $\mathbb{b} = \begin{bmatrix} 6 \\ -3 \end{bmatrix}$

Then $A\mathbb{x} = \mathbb{b}$ says

$$\begin{bmatrix} 3 & 5 \\ 4 & 6 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 6 \\ -3 \end{bmatrix}$$ That is

$$\begin{bmatrix} 3x + 5y \\ 4x + 6y \end{bmatrix} = \begin{bmatrix} 6 \\ -3 \end{bmatrix}$$

Remark: $A\mathbb{x} = \mathbb{b}$ looks a lot like

matrix equation

$$7x = 21 \leftarrow$$ real equation

In the real equation,

$$7^{-1} 7x = 7^{-1} \cdot 21$$
$$1x = \frac{21}{7}$$
$$x = 3$$

would be the way to solve this:

Matrices:

$$A\mathbb{x} = \mathbb{b}$$
$$A^{-1}A\mathbb{x} = A^{-1}\mathbb{b}$$
$$I\mathbb{x} = A^{-1}\mathbb{b}$$

$\rightarrow \mathbb{x} = A^{-1}\mathbb{b}$ That is

$$\begin{bmatrix} x \\ y \end{bmatrix} = A^{-1} \begin{bmatrix} 6 \\ -3 \end{bmatrix}$$

Remark

What acts as the identity matrix?

$$I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \qquad \text{for} \quad \overset{2\times 2}{\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}} \overset{2\times 3}{\begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix}} = \overset{2\times 3}{\begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix}}$$

$$I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad \overset{2\times 3}{\begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix}} \overset{3\times 3}{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}} = \overset{2\times 2}{\begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix}}$$