2.4 Average rate of change of a function

ex: At 9:00 am the temp was $59°F$.

At 12:00 the temp was $77°F$.

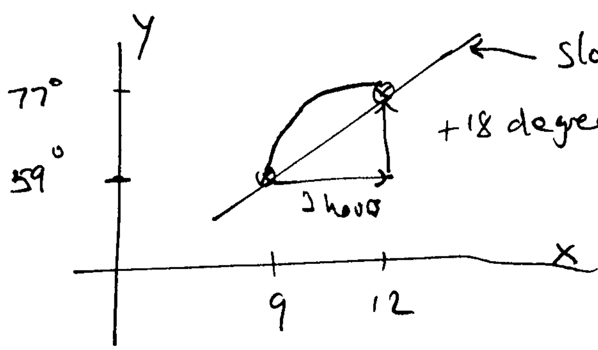
What is the average rate of change of the temperature in degrees per hour?

time	temp.
9	59 degrees
12	77 degrees

$12-9 = 3$ hours

$77-59 = 18$ degrees

$$\frac{77 \text{ deg} - 59 \text{ deg}}{12 \text{ hr} - 9 \text{ hr}} = \frac{+18 \text{ degrees}}{3 \text{ hours}} = 6 \text{ degrees/hour}$$



slope of secant line $= \frac{\Delta y}{\Delta x} = \frac{18 \text{ deg}}{3 \text{ hrs}}$

$= 6$ deg/hr

Suppose the equation of the temperature function has the form $y = f(x)$

where x = time (hours)

and y = temp. (degrees)

Then the average rate of change $= \frac{\text{change in } y}{\text{chang in } x} = \frac{f(12)-f(9)}{12-9} = \frac{77-59}{12-9}$

$= 6$ deg/hr

2.4 14) $f(x) = x + x^4$ between $x = -1$ and $x = 3$

$$\text{avg rate of change} = \frac{f(b) - f(a)}{b - a}$$

x	$f(x)$
-1	$f(-1) = (-1) + (-1)^4 = 0$
3	$f(3) = 3 + 3^4 = 84$

$$\frac{f(3) - f(-1)}{3 - (-1)} = \frac{84 - 0}{4} = \frac{84}{4} = 21$$

16) $f(x) = 4 - x^2$ between $x = 1$ and $x = 1 + h$

x	$f(x) = 4 - x^2$
1	$f(1) = 4 - 1^2 = 3$
$1+h$	$f(1+h) = 4 - (1+h)^2 = 4 - (1 + 2h + h^2)$ $= 4 - 1 - 2h - h^2 = 3 - 2h - h^2$

$$\frac{f(b) - f(a)}{b - a} = \frac{f(1+h) - f(1)}{(1+h) - 1} = \frac{\cancel{3} - 2h - h^2 - \cancel{3}}{\cancel{1} + h - \cancel{1}} = \frac{-2h - h^2}{h}$$

$$= \frac{\cancel{h}(-2 - h)}{\cancel{h}} = -2 - h$$

an expression which depends on h

2.4 20) [Both point involve variables; the algebra is ugly]
Find the average rate of change of

$f(t) = \sqrt{t}$ between $t = a$ and $t = a+h$

Goal: $\begin{pmatrix}\text{avg rate} \\ \text{of change}\end{pmatrix} = \dfrac{f(b) - f(a)}{b - a}$ ← Too complicated to simplify in one fell swoop.

Step 0 $f(a) = \sqrt{a}$

Step 1 $f(a+h) = \sqrt{a+h}$

Step 2 numerator $= f(a+h) - f(a) = \sqrt{a+h} - \sqrt{a}$

Step 3 $\dfrac{\text{numerator}}{\text{denominator}} = \dfrac{\sqrt{a+h} - \sqrt{a}}{(a+h) - a} = \dfrac{\sqrt{a+h} - \sqrt{a}}{h}$

Is there another way to write this? Let's rationalize the numerator:

$$\frac{\sqrt{a+h} - \sqrt{a}}{h} \cdot \frac{\sqrt{a+h} + \sqrt{a}}{\sqrt{a+h} + \sqrt{a}} = \frac{(\sqrt{a+h})^2 - (\sqrt{a})^2}{h(\sqrt{a+h} + \sqrt{a})}$$

$$= \frac{a+h - a}{h(\sqrt{a+h} + \sqrt{a})} = \frac{h}{h(\sqrt{a+h} + \sqrt{a})} = \frac{1}{\sqrt{a+h} + \sqrt{a}}$$

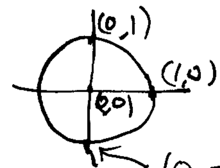
2.5 Transformations of graphs

<u>Remark</u>: We want to account for three "flavors" of transformations:

i) shift ii) reflection iii) stretch

in two directions: x-direction and y-direction.

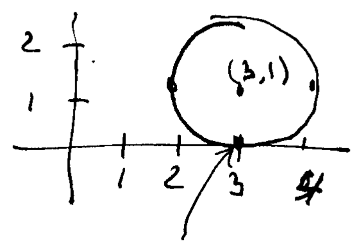
<u>ex</u>: $x^2+y^2=1$



(0,-1) satisfies the equation: $0^2+(-1)^2=1$ ✓

Replace x with $(x-3)$,
Replace y with $(y-1)$:

$$(x-3)^2+(y-1)^2=1$$



$(0+3,\ -1+1)=(3,0)$

<u>Note</u> that $\underbrace{(0+3,-1+1)}_{(3,0)=}$ satisfies the new equation

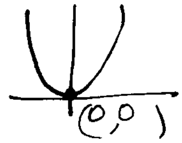
$$(0+3-3)^2+(-1+1-1)^2=0^2+(-1)^2=1$$

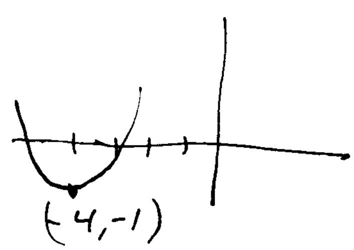
<u>Upshot</u>: Replacing x with $x-3$, and y with $y-1$

<u>shifts</u> a graph 3 units in the positive x-direction and 2 units in the positive y-direction.

<u>ex</u>: $y=x^2$

is satisfied by $(0,0)$





$$y+1=(x+4)^2$$

is satisfied by $(-4,-1)$

Equivalent equation: $y=(x+4)^2-1$

[to be continued]

Two families of useful identities

Binomial expansion formulas

Pascal's triangle:

$$\begin{array}{ccccccccccc} & & & & & 1 & & & & & \\ & & & & 1 & & 1 & & & & \\ & & & 1 & & 2 & & 1 & & & \\ & & 1 & & 3 & & 3 & & 1 & & \\ & 1 & & 4 & & 6 & & 4 & & 1 & \\ 1 & & 5 & & 10 & & 10 & & 5 & & 1 \\ & & & & & \vdots & & & & & \end{array}$$

$$(a+b)^0 = 1$$
$$(a+b)^1 = a+b$$
$$(a+b)^2 = a^2+2ab+b^2$$
$$(a+b)^3 = a^3+3a^2b+3ab^2+b^3$$
$$(a+b)^4 = a^4+4a^3b+6a^2b^2+4ab^3+b^4$$

Difference of n^{th} powers

$$(a-b)(a+b) = a^2-b^2$$
$$(a-b)(a^2+ab+b^2) = a^3-b^3$$
$$(a-b)(a^3+a^2b+ab^2+b^3) = a^4-b^4$$
$$(a-b)(a^4+a^3b+a^2b^2+ab^3+b^4) = a^5-a^5$$
$$\vdots$$

Replace b with $-b$:
(for n odd)

$$(a+b)(a^2-ab+b^2) = a^3+b^3$$
$$(a+b)(a^4-a^3b+a^2b^2-ab^3+b^4) = a^5+b^5$$