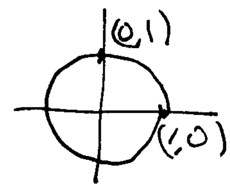
Math 141 Wed 4 Feb 2015

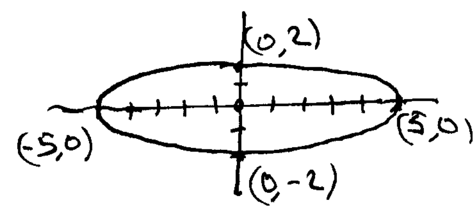
2.5 Transformations of graphs

examples: $x^2 + y^2 = 1$



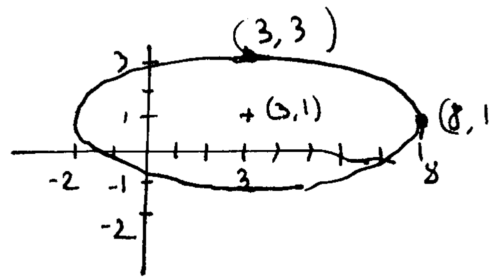
ex: Replace x with $\frac{x}{5}$ and replace y with $\frac{y}{2}$.

$$\left(\frac{x}{5}\right)^2 + \left(\frac{y}{2}\right)^2 = 1 \quad \text{or} \quad \frac{x^2}{25} + \frac{y^2}{4} = 1$$



ex: Now in the previous example Replace x with $x-3$ and replace y with $y-1$.

$$\frac{(x-3)^2}{25} + \frac{(y-1)^2}{4} = 1$$



Note: $(x,y) = (8,1)$ and

$(x,y) = (0+3,\ 2+1) = (3,3)$

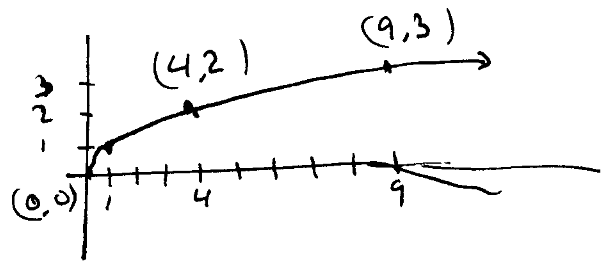
satisfy the new equation

check: $\frac{(8-3)^2}{25} + \frac{(1-1)^2}{4} = \frac{25}{25} + \frac{0}{4} = 1$ ✓

Likewise: $\frac{(3-3)^2}{25} + \frac{(3-1)^2}{4} = 0 + \frac{4}{4} = 1$ ✓

ex: Graph $y = \sqrt{x}$

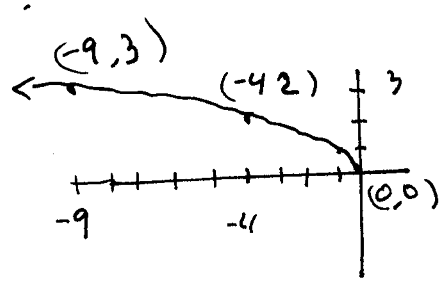
x	$\sqrt{x}$
0	0
1	1
4	2
9	3



Domain = $[0, \infty)$
Range = $[0, \infty)$

Replace x with $-x$.

$y = \sqrt{-x}$



Domain = $(-\infty, 0]$
Range = $[0, \infty)$

x	$\sqrt{-x}$
0	0
-1	1
-4	2
-9	3

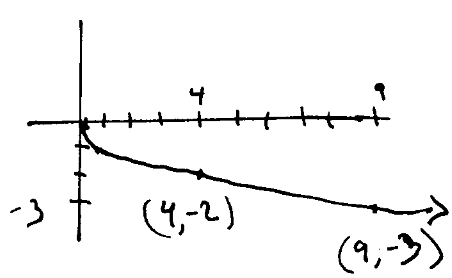
check: If $x = -4$
$y = \sqrt{-(-4)} = \sqrt{4} = 2$

check: If $x = -9$
$y = \sqrt{-(-9)} = \sqrt{9} = 3$

ex: Now take $y = \sqrt{x}$ but replace y with $-y$.

$-y = \sqrt{x}$ which has an equivalent equation
$y = -\sqrt{x}$

x	$-\sqrt{x}$
0	0
1	1
4	2
9	3



Domain = $[0, \infty)$

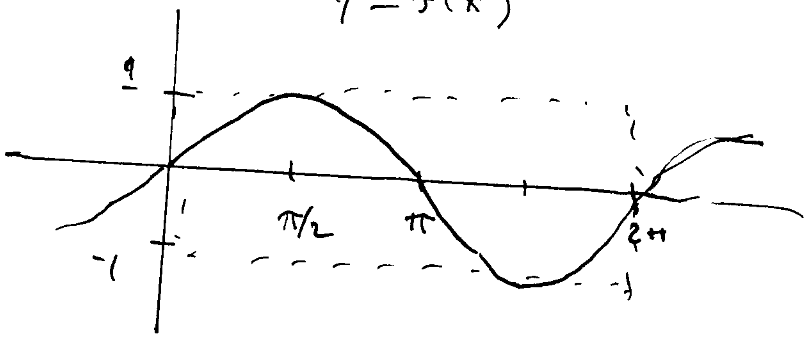
Range = $(-\infty, 0]$

Summary

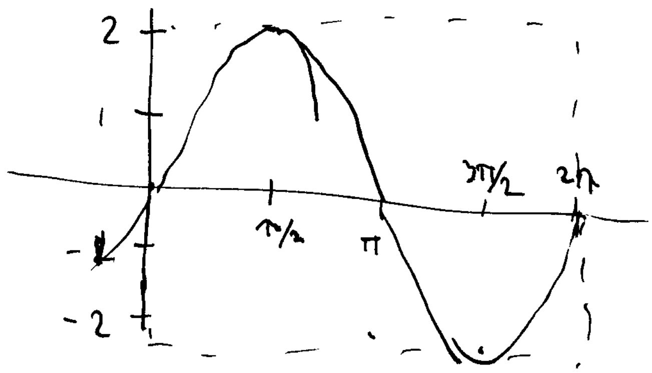
Replacing x with:	Has this effect on the graph
$x-3$	shifting right 3
$\frac{x}{5}$	stretching horizontally by a factor of 5
$-x$	reflecting the x-coordinates (across the y-axis)

Replacing y with	Has this effect
$y-1$	shifts up 1
$\frac{y}{2}$	Stretch vertically by a factor of 2
$-y$	reflecting the y-coordinate (across the x-axis)

$y = f(x)$

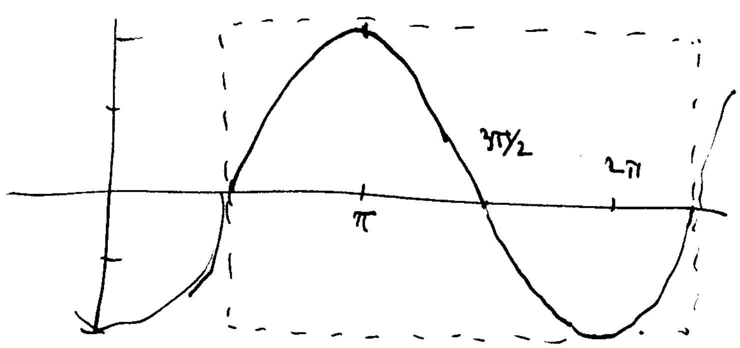


Replace y with $\frac{y}{2}$: we $\frac{y}{2} = f(x)$ or $y = 2f(x)$



$y = 2f(x)$

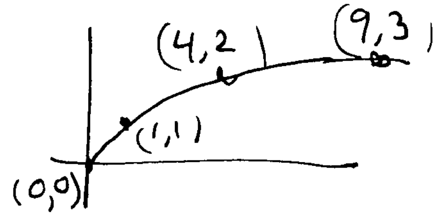
In addition, now replace x with $x - \frac{\pi}{2}$.



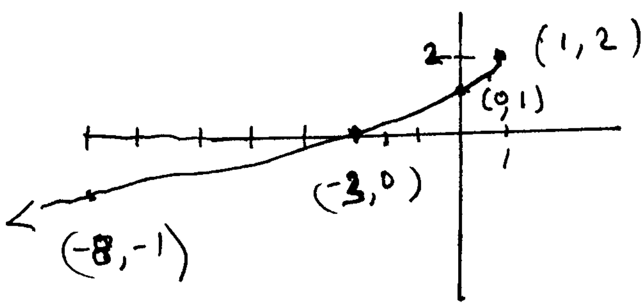
$y = 2f(x - \frac{\pi}{2})$

ex: [modify a graph $\Rightarrow$ modified equation]

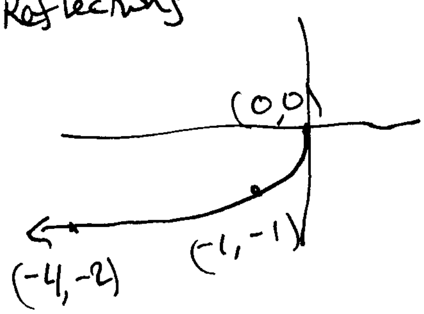
Given that $\boxed{y = \sqrt{x}}$ has this graph



What is the equation of this shifted and reflected graph?



Step 1 Reflections



- Reflect across y-axis
- Reflect across x-axis

corresponding equation:

$$-y = \sqrt{-x}$$

solved for y, this is equivalent to

$$\boxed{y = -\sqrt{-x}}$$

Step 2 Shifts:

- shift right 1
- shift up 2

corresponding equation

This gives us the graph we want (see above)

$$(y-2) = -\sqrt{-(x-1)}$$

or $y = 2 - \sqrt{-(x-1)}$ or ~~y = 2~~

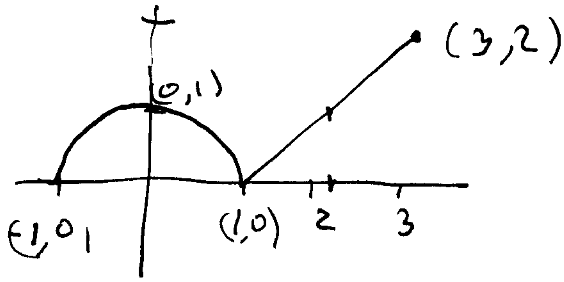
or $\boxed{y = 2 - \sqrt{1-x}}$

OR $y = 2 - \sqrt{-x+1}$

x	$2-\sqrt{1-x}$
1	2
0	1
-3	0
-8	$2-\sqrt{1-(-8)} = 2-\sqrt{9} = 2-3 = -1$ ✓

Suppose it is given that

ex: ~~Here is~~ the graph of $y = g(x)$ looks like this:



Domain = $[-1, 3]$

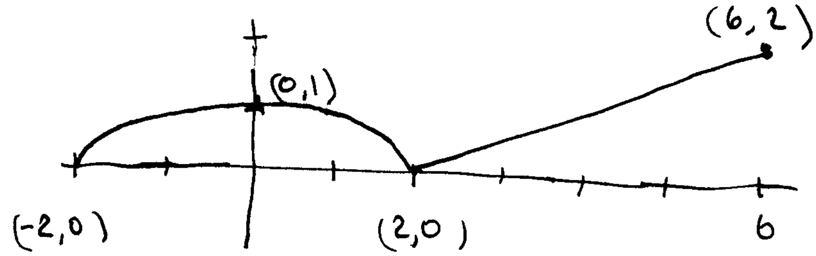
Range = $[0, 2]$

Now sketch the graph of $y = 2\,g\left(\frac{x-2}{2}\right)$

Domain = $[-2, 6]$

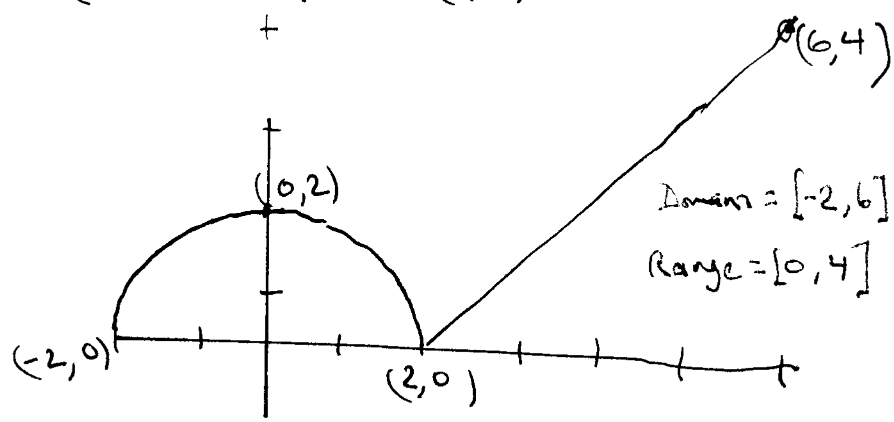
Range = $[0, 2]$

Step 1 Graph $y = g\left(\frac{x}{2}\right)$



Step 2 Graph $y = 2g\left(\frac{x}{2}\right)$

or $\frac{y}{2} = g\left(\frac{x}{2}\right)$

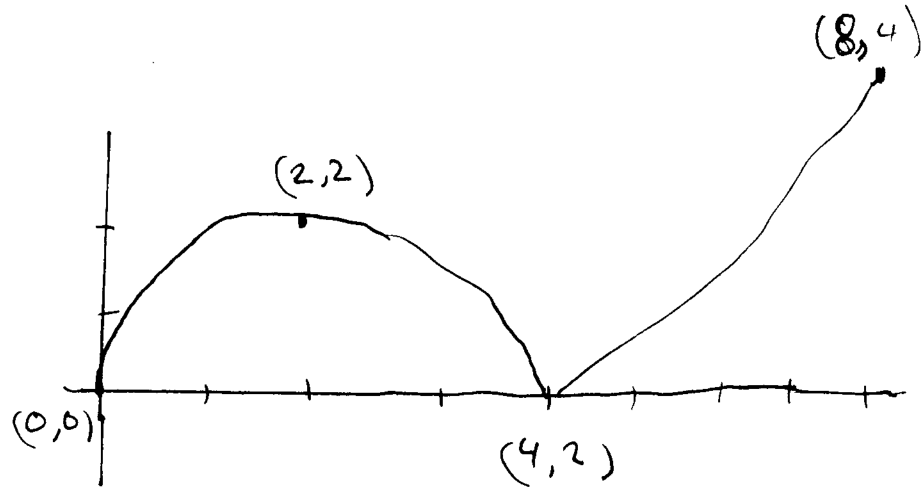


Domain = $[-2, 6]$

Range = $[0, 4]$

Step 3 Graph $y = 2g\left(\frac{x-2}{2}\right)$

or $\frac{y}{2} = g\left(\frac{x-2}{2}\right)$



Domain = $[0, 8]$

Range = $[0, 4]$

ex: A bit more on this example

The original equation of the graph seems to be

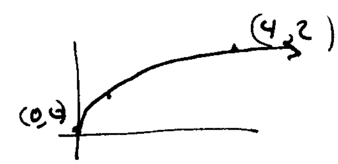
$$y = g(x) = \begin{cases} \sqrt{1-x^2} & \text{if } -1 \le x \le 1 \\ x-1 & \text{if } 1 \le x \le 3 \end{cases}$$

So the final graph would seem to have as its equation

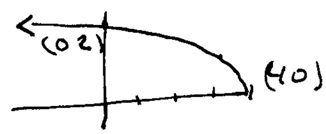
$$y = 2\,g\left(\frac{x-2}{2}\right) \quad , \text{ that is}$$

$$y = \begin{cases} 2\sqrt{1-\left[\frac{(x-2)}{2}\right]^2} & \text{if } 0 \le x \le 4 \\ 2\left[\frac{(x-2)}{2} - 1\right] & \text{if } 4 \le x \le 8 \end{cases}$$

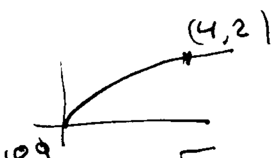
ex: Given that $y = \sqrt{x}$ has this graph



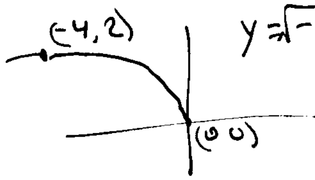
Find an equation for this graph



method 1: Reflect, then shift right 4



$y = \sqrt{x}$



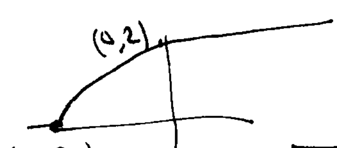
$y = \sqrt{-x}$

$\boxed{y = \sqrt{-(x-4)}}$

Are these two answers equivalent?
Yes: $\sqrt{-(x-4)} = \sqrt{-x+4}$

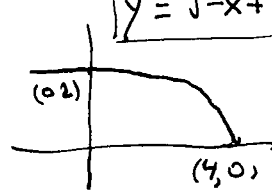
method 2: Shift left 4, then reflect

$y = \sqrt{x}$



$y = \sqrt{x+4}$

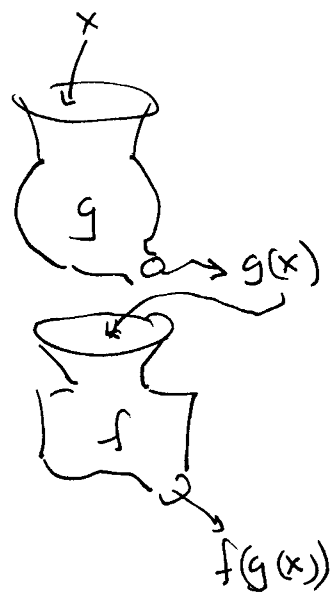
$\boxed{y = \sqrt{-x+4}}$



2.6 Composition of functions

ex: Suppose that $f(x) = x^2 + 5x$
and $g(x) = x - 3$

Defn: Given functions f and g, the composition of f and g, denoted $f \circ g$ is defined by $(f \circ g)(x) = f(g(x))$



a) Find $(f \circ g)(x)$.

ex: (contd) $f(g(x)) = f(x-3)$

$$= (x-3)^2 + 5(x-3)$$

b) Find $(g \circ f)(x) = g(f(x))$

$$= g(x^2 + 5x)$$
$$= x^2 + 5x - 3$$

c) Find $(g \circ g)(x) = g(g(x)) = g(x-3) = (x-3) - 3$
$$= x - 6$$

d) Find $(f \circ f)(x) = f(f(x))$

$$= f(x^2 + 5x)$$
$$= (x^2+5x)^2 + 5(x^2+5x)$$

Quiz likely Monday

2.4 Avg rate of change

2.5 Transformations

2.3 Domain & range from graph

2.6 Composition of functions