Warmups

2.6 44) $f(x) = \frac{2}{x}$ $g(x) = \frac{x}{x+2}$

a) Find $(f \circ g)(x)$, and the domain of $f \circ g$.

$$(f \circ g)(x) = f(g(x)) = f\left(\frac{x}{x+2}\right) = \frac{2}{\left(\frac{x}{x+2}\right)}$$

$$= 2 \div \frac{x}{x+2} = 2 \cdot \frac{x+2}{x} = \frac{2x+4}{x} = \frac{2x}{x} + \frac{4}{x}$$

$$= 2 + \frac{4}{x}$$

Domain $= \{ x \mid x \neq -2 \text{ and } x \neq 0 \}$

b) Find $(g \circ f)(x)$ and the domain of $g \circ f$.

$$g(f(x)) = g\left(\frac{2}{x}\right) = \frac{\frac{2}{x}}{\frac{2}{x}+2} \cdot \frac{x}{x} = \frac{2}{2+2x} = \frac{2 \cdot 1}{2(1+x)}$$

$$= \frac{1}{1+x}$$

domain of $g \circ f = \{ x \mid x \neq 0 \text{ and } x = -1 \}$

49) $F(x) = (x-9)^5$ Write F as $f \circ g$ for some f, g.

Let $g(x) = x - 9$ and $f(x) = x^5$

Crazy answer: Let $g(x) = \frac{x}{17}$ and $f(x) = (17x - 9)^5$

check the crazy answer: $f(g(x)) = f\left(\frac{x}{17}\right) = \left[17 \cdot \left(\frac{x}{17}\right) - 9\right]^5$

$$= (x-9)^5$$

"Trivial" answer: Let $f(x) = x$

and $g(x) = (x-9)^5$.

2.7 One-to-one functions and their inverses

<u>Defn</u>: A function f is <u>one-to-one</u> if

$$f(x_1) \neq f(x_2) \text{ whenever } x_1 \neq x_2, \text{ for } \underline{\text{all}}\ x_1, x_2 \text{ in the domain of } f.$$

Logically <u>Equivalent</u>: That is, if $x_1 \neq x_2$ then $f(x_1) \neq f(x_2)$

<u>Equivalent</u> If $f(x_1) = f(x_2)$ then $x_1 = x_2$.

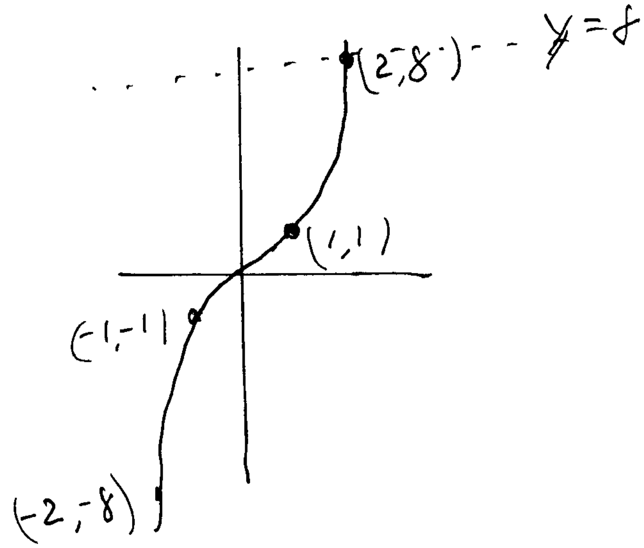
<u>Graphical interpretation</u>: The graph <u>passes</u> the <u>horizontal</u> line test.

<u>Tabular interpretation</u>. There are no repeated numbers in the <u>second</u> column.

<u>ex</u>: (one-to-one) $y = x^3$

x	x^3
-3	-27
-2	-8
-1	-1
0	0
1	1
2	8
3	27
4	64 ← ignore this

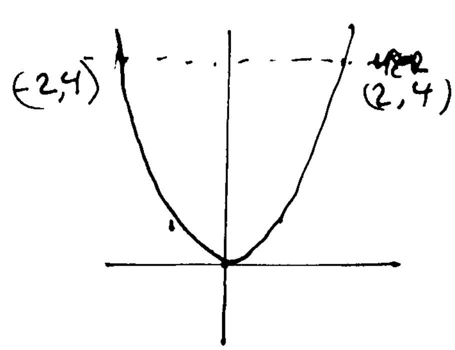
No repeats in 2^{nd} column



<u>ex</u>: (Not one-to-one) $y = x^2$

x	x^2
-2	4
-1	1
0	0
1	1
2	4

Repeated "4"s



<u>Defn</u>: Let f be a one-to-one function.

The <u>inverse function</u> f^{-1} (<u>terrible</u> notation, not to be confused with "reciprocal") is defined by:

$$f^{-1}(y) = x \quad \text{means} \quad f(x) = y.$$

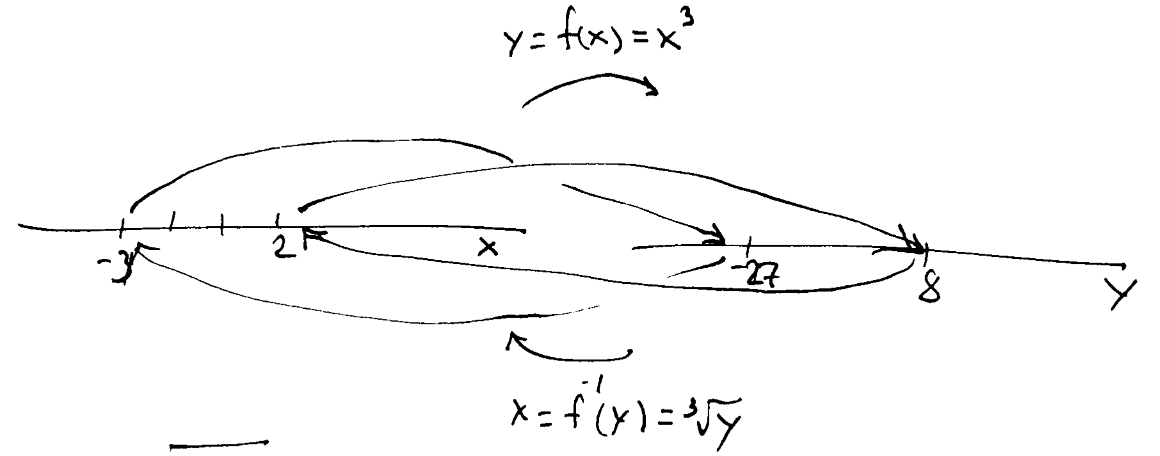
<u>ex</u>: ~~$f^{-1}(y) = \sqrt[3]{x}$ means $f(x) =$~~

$$\sqrt[3]{y} = x \quad \text{means} \quad x^3 = y$$

<u>Inverse function property</u>: If f is one-to-one and f^{-1} is its inverse

1. $f^{-1}(f(x)) = x$
2. $f(f^{-1}(x)) = x$

<u>ex</u>.

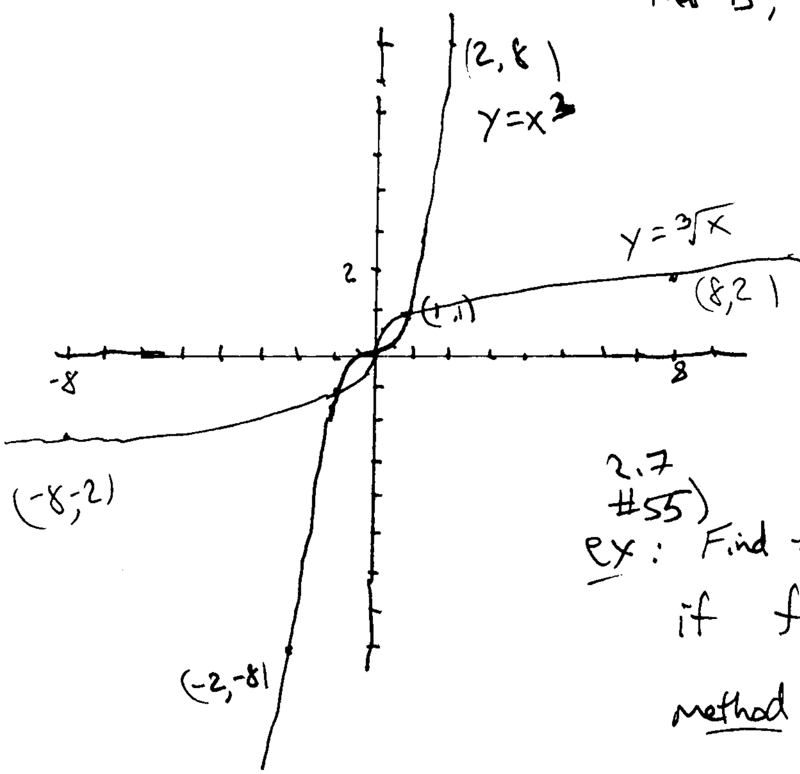


<u>ex</u>: ① $\sqrt[3]{x^3} = x$

② $(\sqrt[3]{y})^3 = y$

Graph of inverse functions : swap x- and y-coordinates

that is, reflect across the $y=x$ line.



2.7 #55)

ex : Find the ~~inver~~ equation of the inverse of f

if $f(x) = 4 + \sqrt[3]{x}$.

method : $y = 4 + \sqrt[3]{x}$ Solve for x:

$y - 4 = -4 + 4 + \sqrt[3]{x}$

$y - 4 = \sqrt[3]{x}$

$(y-4)^3 = (\sqrt[3]{x})^3$

$(y-4)^3 = x$ Now swap x and y:

$(x-4)^3 = y$ so

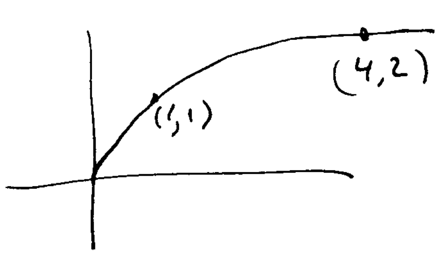
$f^{-1}(x) = (x-4)^3$.

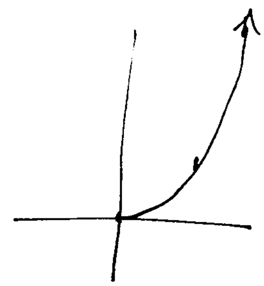
What about square roots? Or arcsine?

<u>Defn</u>: $y = \sqrt{x}$ means ① $y^2 = x$ (i.e. y is a square root of x)

and ② $y \geq 0$. (i.e. y is the <u>positive</u> square root)

$f(x) = \sqrt{x}$





$$y = f^{-1}(x) = \begin{cases} x^2 & \text{if } x \geq 0 \\ \text{undefined} & \text{if } x < 0 \end{cases}$$

2.7

54) $f(x) = \sqrt{2x-1}$ Find the equation of $f^{-1}(x)$.

$y = \sqrt{2x-1}$ This means:

① $y^2 = 2x - 1$ and ② $y \geq 0$

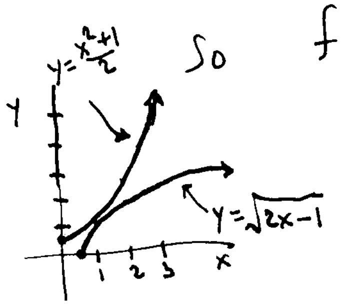
Solve for x: $y^2 + 1 = 2x - 1 + 1$ and $y \geq 0$

$y^2 + 1 = 2x$ and $y \geq 0$

$\frac{y^2+1}{2} = \frac{2x}{2}$ and $y \geq 0$

$\frac{y^2+1}{2} = x$ and $y \geq 0$

Swap x and y: $\frac{x^2+1}{2} = y$ and $x \geq 0$



So $$f^{-1}(x) = \begin{cases} \frac{x^2+1}{2} & \text{if } x \geq 0 \\ \text{undefined} & \text{if } x < 0 \end{cases}$$

3.1 Quadratic functions

Vocabulary:

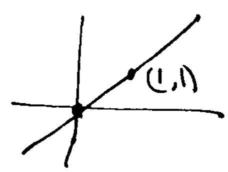
degree	name	example
0	~~fcn~~ constant function	$f(x) = 5$
1	linear function	$f(x) = 2x - 1$
2	quadratic function	$f(x) = x^2 - 8x + 15 = (x-3)(x-5)$
3	cubic function	$f(x) = x^3 - 4x = x(x-2)(x+2)$
4	quartic function (etc.)	$f(x) = -2x^4 + 7x^3 + 2x^2 - \frac{1}{2}x + 17$

or $g(x) = x^4 - 16$
$= (x^2 - 4)(x^2 + 4)$
$= (x-2)(x+2)(x^2+4)$

or $h(x) = (x-2)^2(x+2)^2$
$= (x^2-4)^2 = x^4 - 8x^2 + 16$

Review: Linear functions

$y = x$

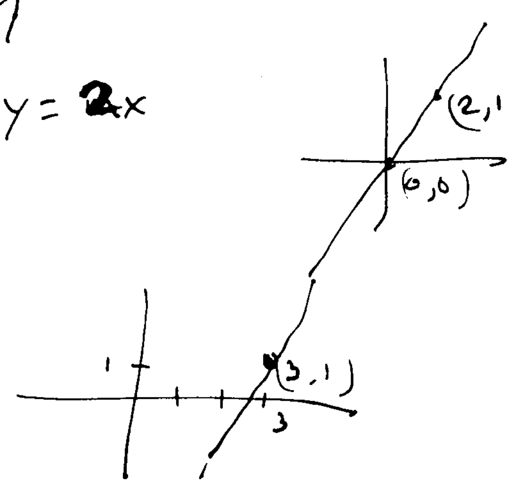


~~multiply~~

Replace x with $2x$ $y = 2x$

Replace x with $(x-3)$
and y with $(y-1)$

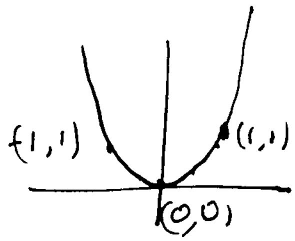
$y - 1 = 2(x-3)$



Notice this is point-slope form of a line.

Review: Quadratic function

$y = x^2$

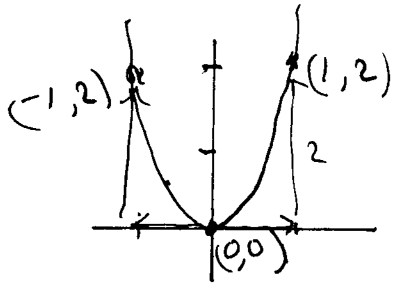


multiply by 2

$y = 2x^2$

note: we Replace y with $\frac{y}{2}$

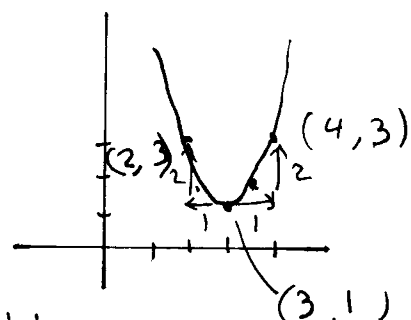
$\frac{y}{2} = x^2$ or $y = 2x^2$



Replace x with $x-3$ and y with $y-1$:

$y - 1 = 2(x-3)^2$

or $y = 2(x-3)^2 + 1$



Standard form of a quadratic function
(or vertex form)

$$\boxed{y = a(x-h)^2 + k}$$

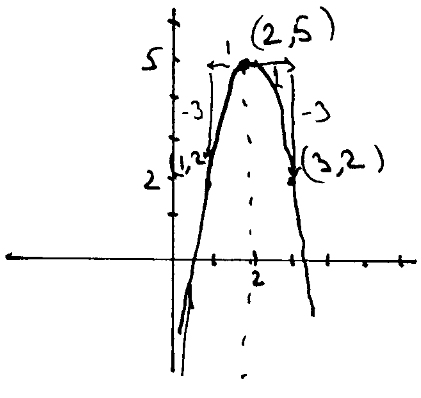
$(h,k) =$ vertex

open up if $a > 0$

ex: Graph $y = -3(x-2)^2 + 5$

$a = -3$
$h = 2$
$k = 5$

vertex $= (2, 5)$
open down

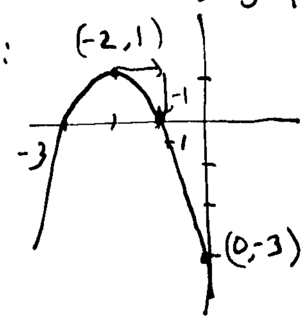


x	y
1	2
2	5
3	2

if $x=3$, $y = -3(3-2)^2 + 5$
$= -3 + 5 = 2$

Given this graph, find the equation.

ex:



$y = -1\cdot(x+2)^2 + 1$

$y = -(x+2)^2 + 1$

That is, $a = -1$
$h = -2$
$k = 1$