3.1 Quadratic function

Summary: Equations of lines

$\boxed{y = mx + b}$ ~~point~~ slope-intercept
m = slope, $(0, b)$ = y-intercept

$\boxed{y - y_1 = m(x - x_1)}$ point-slope
m = slope (x_1, y_1) = point.

$\boxed{ax + by = c}$ general form

Summary: Quadratic function equations

$\boxed{f(x) = a(x-h)^2 + k}$ Standard form or vertex form
(h, k) = vertex
$a > 0$, open up.

$\boxed{f(x) = ax^2 + bx + c}$ General form
$(0, c)$ = y-intercept

Fact: $\boxed{\begin{array}{l} a = a \\ h = -\frac{b}{2a} \\ k = f(h) \end{array}}$

$\boxed{f(x) = a(x - r_1)(x - r_2)}$ x-intercept form

$(r_1, 0)$ and $(r_2, 0)$ will be the x-intercepts of the graph.

$\boxed{h = \frac{r_1 + r_2}{2}}$

3.1

30) $g(x) = 2x^2 + 8x + 11$

a) Put into std. form

b) Graph

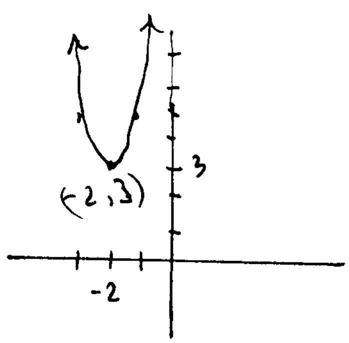
c) Find the min

<u>Method</u>: Complete the square:

$$g(x) = (2x^2 + 8x) + 11$$
$$= 2(x^2 + 4x \quad) + 11$$
$$= 2(x^2 + 4x + 4) + 11 - 8$$
$$g(x) = 2(x+2)^2 + 3$$

So $(h,k) = (-2, 3)$

$a = 2$ open up



y-int $= (0, 11)$

Min value $= 3$

domain $= (-\infty, \infty)$ range $= [3, \infty)$

<u>More generally</u>: $y = ax^2 + bx + c$

$$y = (ax^2 + bx) + c$$
$$y = a\left(x^2 + \frac{b}{a}x \quad \right) + c$$
$$y = a\left(x^2 + \frac{b}{a}x + \frac{b^2}{4a^2}\right) + c - \frac{b^2}{4a}$$
$$y = a\left(x + \frac{b}{2a}\right)^2 + \frac{4ac - b^2}{4a}$$

has the form

$$y = a(x-h)^2 + k$$

<u>Scratch</u>

i) $\frac{1}{2} \cdot \frac{b}{a} = \frac{b}{2a}$

ii) $\left(\frac{b}{2a}\right)^2 = \frac{b^2}{4a^2}$

where

$$a = a$$
$$h = -\frac{b}{2a}$$
$$k = \frac{4ac - b^2}{4a}$$

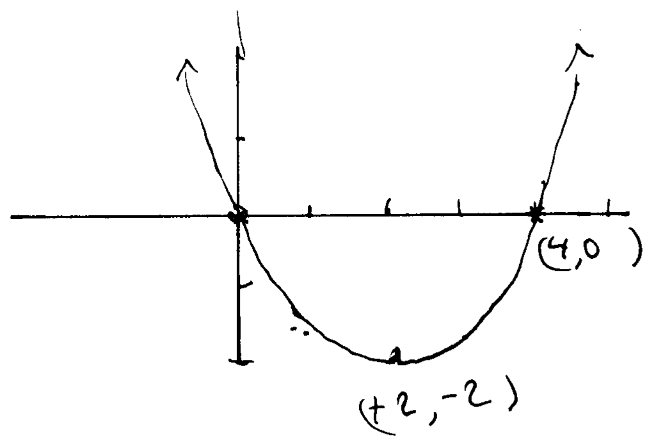
<u>Remark</u>: We are now justified in taking

$$h = -\frac{b}{2a}$$

graph → equation for quadratic functions

ex: [given vertex and another point]

Find the equation. Use standard or vertex form



$f(x) = a(x-h)^2 + k$

where $\boxed{(h,k) = (2,-2)}$

All that's left is to find a.

Use that $(x,y) = (4,0)$ satisfies the equation

$$f(x) = a(x-2)^2 - 2$$

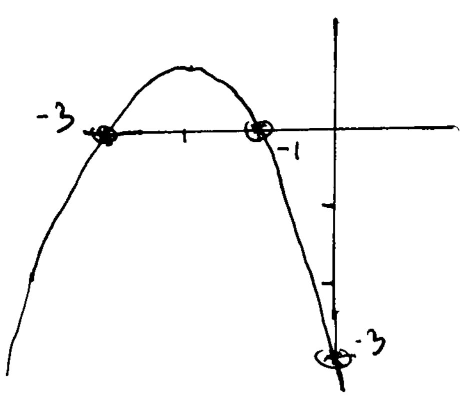
Plug in:

This is a true equation: $0 = a(4-2)^2 - 2$

$0 = 4a - 2 \Rightarrow 4a = 2 \Rightarrow \boxed{a = \frac{1}{2}}$

Answer: $\boxed{f(x) = \frac{1}{2}(x-2)^2 - 2}$

ex: [Given x-intercepts and another point]



$f(x) = a(x+3)(x+1)$

so $f(-3) = a(-3+3)(-3+1) = 0$

and $f(-1) = a(-1+3)(-1+1) = 0$

Use $(0,-3)$ satisfies the equation.

$-3 = a(0+3)(0+1)$

$\Rightarrow -3 = 3a \Rightarrow a = -1$

$$\boxed{f(x) = -(x+3)(x+1)}$$

3.2 Polynomial functions

Main idea: If P is a polynomial and c is a real number, the following are equivalent.

(1) c is a zero of P (that is, if $x=c$ is an input to the function, $y=0$ is the output)

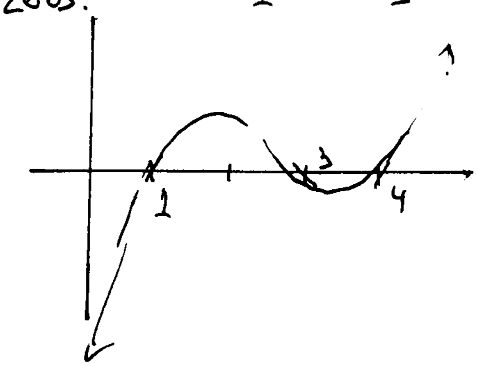
(2) $x=c$ is a solution of the equation $P(x)=0$.

(3) $x-c$ is a factor of $P(x)$.

(4) c is a x-intercept of the graph of P.

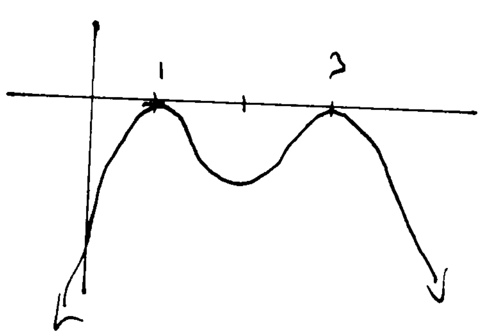
ex Graph $P(x) = (x-1)(x-3)(x-4) = x^3 - \text{(other terms)}$

zeros: 1 3 4



ex: Graph $P(x) = -\frac{1}{2}(x-1)^2(x-3)^2 = -\frac{1}{2}(x-1)(x-1)(x-3)(x-3) = -\frac{1}{2}x^4 + \text{(other terms)}$

zeros: 1 3 | 1 1 3 3



we say that 1 and 3 are "zeros of multiplicity 2".

M141 2/11/15

Factor. Use the factors to sketch the graph

5

3.2 28) $P(x) = x^3 + 2x^2 - 8x$

$= x(x^2 + 2x - 8) = x(x+4)(x-2)$

zeros: 0   -4   2