Warmup

Find the difference quotient.

41) $f(x) = 3 - 5x + 4x^2$

$f(a) = 3 - 5a + 4a^2$

$f(a+h) = 3 - 5(a+h) + 4(a+h)^2$

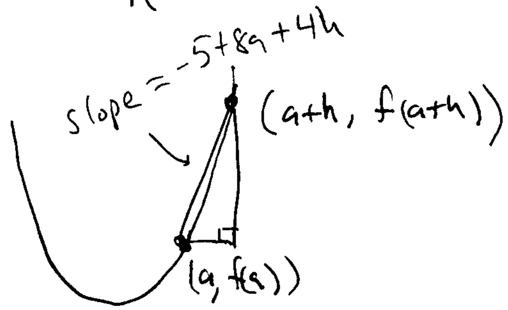
$= 3 - 5a - 5h + 4(a^2 + 2ah + h^2)$

$= 3 - 5a - 5h + 4a^2 + 8ah + 4h^2$

$f(a+h) - f(a) = 3 - 5a - 5h + 4a^2 + 8ah + 4h^2$
$- 3 + 5a - 4a^2$

$= -5h + 8ah + 4h^2 = h(-5 + 8a + 4h)$

$$\frac{f(a+h) - f(a)}{h} = \frac{\cancel{h}(-5 + 8a + 4h)}{\cancel{h}} = \boxed{-5 + 8a + 4h}$$



More warmups

Find the zeros of

3.4 84) $P(x) = 8x^5 - 14x^4 - 22x^3 + 57x^2 - 35x + 6$

How many could we have? Five.

Use Rational zeros theorem.

$p: \pm 1, 2, 3, 6$

$q: \pm 1, 2, 4, 8$

$\frac{p}{q}: \pm 1, 2, 3, 6, \frac{1}{2}, \frac{3}{2}, \frac{1}{4}, \frac{3}{4}, \frac{1}{8}, \frac{3}{8}$.

Descartes' Rule of signs:

$P(x)$ has 4 variations in sign $\Rightarrow$ 4 or 2 or No positive zeros.

$P(-x) = -8x^5 - 14x^4 + 22x^3 + 57x^2 + 35x + 6$

has one variation in sign $\Rightarrow$ there is exactly one negative zero.

From a portion of the graph, it looks like 1 and $3/4$ might be rational zeros, as well as an irrational(?) zero about at $x = 0.3$.

Is $x=1$ a zero?

1	8	-14	-22	57	-35	6
		8	-6	-28	29	-6
	8	-6	-28	29	-6	0 = P(1)

so $(x-1)$ is a factor

Try $x = 3/4$

3/4		6	0	-21	6
	8	0	-28	8	0

So $(x - 3/4)$ is also a factor

we found:

$$P(x) = (x-1)(8x^4 - 6x^3 - 28x^2 + 29x - 6)$$

$$= (x-1)(x - \tfrac{3}{4})(8x^3 - 28x + 8)$$

Rearrange factors

$$= (x-1)(4x-3)(2x^3 - 7x + 2)$$

ex (cont'd)

Three more zeros to find, which will solve $2x^3-7x+2=0$

Remaing rational zeros of this polynomial?

$p: \pm 1 \quad \pm 2$

$q: \pm 1 \quad \pm 2$ $\Rightarrow \frac{p}{q}: \pm 1, \pm 2 \quad \pm \frac{1}{2}$

missing x^2 term.

Try -2:

$$\begin{array}{r|rrrr} -2 & 2 & 0 & -7 & 2 \\ & & -4 & 8 & -2 \\ \hline & 2 & -4 & 1 & 0 \end{array}$$

so $(x+2)$ is a factor

Now: $P(x) = (x-1)(4x-3)(x+2)(2x^2-4x+1)$

zeros: $1 \quad 3/4 \quad -2 \quad ? \quad ?$

To find the last two zeros:

Solve: $0 = 2x^2-4x+1$ $\qquad x = \dfrac{4 \pm \sqrt{(-4)^2-4(2)(1)}}{2(2)}$

$$x = \frac{4 \pm \sqrt{8}}{4} = \frac{4 \pm 2\sqrt{2}}{4} = \frac{2(2 \pm \sqrt{2})}{4}$$

$$= \frac{2 \pm \sqrt{2}}{2} = \frac{2}{2} \pm \frac{\sqrt{2}}{2} = 1 \pm \frac{\sqrt{2}}{2}$$

$$\approx 1 \pm .707 = \begin{cases} 1.707 & \text{or} \\ 0.293 \end{cases}$$

Answer: The zeros are $\boxed{1, \ 3/4, \ -2, \ 1+\frac{\sqrt{2}}{2}, \ 1-\frac{\sqrt{2}}{2}}$

TRY THIS! Find the zeros of

3.4 47) $P(x) = x^3 + 4x^2 + 3x - 2$

Descartes' Rule of Signs:
1 positive zero

$P(-x) = -x^3 + 4x^2 - 3x - 2$

2 or no negative zeros

Possible rational zeros: $p: \pm 1 \ \pm 2$; $q: \pm 1$ $\Rightarrow$ $\frac{p}{q}: \pm 1 \ \pm 2$

From the graph, $x = -2$ <u>might</u> be a zero. Check.

$$\begin{array}{r|rrr|r} -2 & 1 & 4 & 3 & -2 \\ & & -2 & -4 & 2 \\ \hline & 1 & 2 & -1 & 0 \end{array}$$

$= P(-2)$ so $(x+2)$ is a factor

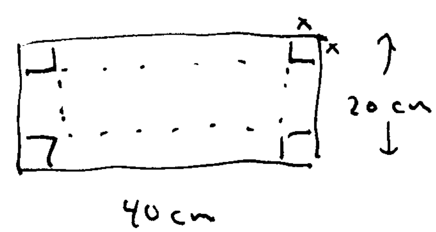
$P(x) = (x+2)(x^2+2x-1)$

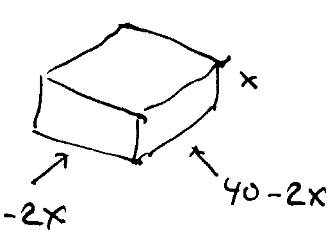
Solve $0 = x^2+2x-1$ $\Rightarrow$

$$\begin{aligned} x^2+2x &= 1 \\ x^2+2x+1 &= 1+1 \\ (x+1)^2 &= 2 \\ x+1 &= \pm\sqrt{2} \\ x &= -1 \pm \sqrt{2} \end{aligned}$$

$\therefore$ zeros: $\boxed{-1+\sqrt{2},\ -1-\sqrt{2},\ -2}$

3.4 102)





Express a function to represent volume, given x = dimensions of corners cut out.

Volume $= x(20-2x)(40-2x)$, domain $= [0, 10]$

$= x(4x^2 - 120x + 800$

$= 4x^3 - 120x^2 + 800x$

<u>want</u> volume $= 1500\ cm^3$ so Solve $4x^3 - 120x^2 + 800x = 1500$

or $\frac{4x^3}{4} - \frac{120x^2}{4} + \frac{800x}{4} - \frac{1500}{4} = \frac{0}{4}$

So Solve $x^3 - 30x^2 + 200x - 375 = 0$ where in $0 \le x \le 10$

$$\begin{array}{r|rrrr} 5 & 1 & -30 & 200 & -375 \\ & & 5 & -125 & 375 \\ \hline & 1 & -25 & 75 & 0 \end{array}$$

Let $P(x) = x^3 - 30x^2 + 200x - 375$

By graphing calculator, we see that the two x-intercepts for $0 \le x \le 10$ cm

are $x = \cancel{3.5}$ cm and $x = 5.0$ cm

$3.486 = \frac{25 - 5\sqrt{13}}{2}$

approximately 3.5 cm × 13 cm × 33 cm

5 cm × 10 cm × 30 cm.

3.5 Arithmetic of complex numbers

Defn: $i = \sqrt{-1}$ so $\boxed{i^2 = -1}$ so i is a constant but it's not real constant.

Defn: $a + bi$ where a and b are real numbers is a complex number.

ex: $2+3i$, $\frac{1}{2} + \frac{\sqrt{3}}{2}i$, $-11i = 0 - 11i$, $17 = 17 + 0i$

So $\sqrt{-49} = \sqrt{49}\sqrt{-1} = 7i$

Add, subtract: $(3+2i) + (7+5i) = 3+7+2i+5i = 10+7i$

$(2+8i) - (4-3i) = 2-4+8i+3i = -2+11i$

$= 2+8i-4+3i$

Multiply: $(3i)(5i) = 15i^2 = 15(-1) = -15$

$7i(2+3i) = (7i)2 + (7i)(3i) = 14i + 21i^2$

$= -21 + 14i$

$(2+3i)\cdot(5+i) = 10 + 2i + 15i + 3i^2$

$= 10 + 17i - 3 = 7 + 17i$

Multiplying by a "Conjugate":

use $(A+B)(A-B) = A^2 - B^2$

ex: $(5+12i)(5-12i) = 5^2 - (12i)^2$
$= 25 - 144i^2 = 25 - 144(-1)$
$= 25 + 144 = 169$

Division

ex: $\dfrac{7+17i}{5+i} \cdot \dfrac{5-i}{5-i} = \dfrac{35 - 7i + 85i - 17i^2}{5^2 - i^2}$

$= \dfrac{35 + 78i + 17}{25 - (-1)} = \dfrac{52 + 78i}{26} = \dfrac{52}{26} + \dfrac{78}{26}i$
$= 2 + 3i$

We just said: $(7+17i) \div (5+i) = 2 + 3i$

To check: $(5+i)(2+3i) = 7 + 17i$

Powers of i:

	$i^0 = 1$	$i^4 = 1$	$i^8 = 1$	... $i^{2015} = -i$
$i^{-3} = i$	$i^1 = i$	$i^5 = i$	$i^9 = i$	
$i^{-2} = -1$	$i^2 = -1$	$i^6 = -1$	$i^{10} = -1$	
$i^{-1} = -i$	$i^3 = -i$	$i^7 = -i$	$i^{11} = -i$	

why? $i^{11} = i^4 \cdot i^4 \cdot i^3$
$= (i^2 \cdot i^2) \cdot (i^2 \cdot i^2) \cdot i^2 \cdot i^1$
$= \underbrace{(-1 \cdot -1)}_{1} \cdot \underbrace{(-1 \cdot -1)}_{1} \cdot (-1)\, i = -i$

ex: $i^{-3} = \dfrac{1}{i^3} \cdot \dfrac{i}{i} = \dfrac{i}{i^4} = \dfrac{i}{1} = i$

example of complex arithmetic.

Given $P(x) = x^2 - 2x + 2$

Q: Is $x = 1+i$ a zero of $P(x)$?

$$\begin{array}{c|ccc} 1+i & 1 & -2 & 2 \\ & & 1+i & -2 \\ \hline & 1 & -1+i & 0 \end{array} = P(1+i) \quad \text{Yes}$$

Scratch work

$(1+i)(-1+i)$

$= (i+1)(i-1)$

$= i^2 - 1^2 = -1-1 = -2$

In fact

$$P(x) = [x - (1+i)]\,[x + (-1+i)]$$
$$= (x-1-i)(x-1+i)$$

zeros: $1+i \qquad 1-i$

Why we already knew this:

$$x^2 - 2x + 2 = 0$$
$$x^2 - 2x + 1 = -2+1$$
$$(x-1)^2 = -1$$
$$x - 1 = \pm\sqrt{-1} = \pm i$$
$$x = \underline{1 \pm i}$$

3.6 Complex zeros: Find the zeros, and factor $P(x)$.

8) $P(x) = x^3 + x^2 + x = x(x^2 + x + 1)$

$0 \quad ? \quad ?$

zeros?

Solve $0 = x^2 + x + 1$

Use $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \dfrac{-1 \pm \sqrt{1-4}}{2}$

$= \dfrac{-1 \pm \sqrt{-3}}{2} = \dfrac{-1}{2} \pm \dfrac{\sqrt{3}}{2} i$

so zeros: $0, \ -\frac{1}{2} + \frac{\sqrt{3}}{2} i, \ \frac{-1}{2} - \frac{\sqrt{3}}{2} i$

$$P(x) = x\left[x - \left(-\tfrac{1}{2} + \tfrac{\sqrt{3}}{2}i\right)\right]\left[x - \left(-\tfrac{1}{2} - \tfrac{\sqrt{3}}{2}i\right)\right]$$

$$= x\left(x + \tfrac{1}{2} - \tfrac{\sqrt{3}}{2}i\right)\left(x + \tfrac{1}{2} + \tfrac{\sqrt{3}}{2}i\right)$$

ex: #42) If $P(x)$ has $2i$ and $3i$ as zeros and has <u>real</u> coefficients, what is $P(x)$?

Reason:
By the conjugate zeros theorem, complex zeros must appear as conjugate pairs, that is, if $2i$ is a zero, so is $-2i$; if $3i$ is a zero, so is $-3i$.

zeros: $2i$, $-2i$, $3i$, $-3i$

$$P(x) = (x-2i)(x+2i)(x-3i)(x+3i)$$

$$= (x^2 - 4i^2)\cdot(x^2 - 9i^2)$$

$$= (x^2+4)(x^2+9) = \boxed{x^4 + 13x^2 + 36}$$

- Test 1 on Chap 2 and Chap 3 will be on Monday, March 2.
- Quiz 3 on 3.2–3.5 will be on Wednesday, Feb. 25.